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NONLINEAR MODEL PREDICTIVE CONTROL OF AN EVAPORATOR SYSTEM USING FUZZY-NEURAL OBSERVER

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Abstract
In this paper it is proposed a nonlinear approach to model predictive control that is based on a Takagi– Sugeno (TS) fuzzy model representation of a state observer. An industrial evaporator system is taken as an exemplary process and its prediction model is used in the controller. Accurate nonlinear models of the evaporator system components are described. The final model of the evaporator system in state space implementation is used in model based control. The MPC scheme is based on an explicit use of the predictive model of the system response to obtain the control actions by minimizing a cost function. Optimization objectives in MPC include minimization of the difference between the predicted and desired response trajectories, and the control effort subjected to prescribed constraints. The case study is implemented using MATLAB/Simulink. The simulation results show that the main process variables have good performance and process quality is satisfied.

Keywords: *Model Predictive control, Fuzzy-Neural Modelling, Fuzzy-Neural Observer, Process Control*

Introduction

Model predictive control is a model-based control strategy that has been applied to a large number of industrial processes, where a sequence of future control actions is computed by minimizing an objective function. In the last two decades, linear model predictive control has been well recognized by industry due to its intuitiveness and capability to handle multivariable constraints. However, the extension to nonlinear model based predictive control (NMPC) has not been so successful although a significant amount of research effort has been put into this area.

One of the main obstacles, which blocks NMPC techniques to become widely applicable, is the computational burden associated with the requirement to solve a set of nonlinear differential equations and a nonlinear dynamic optimization problem in real-time. The objective of NMPC is to determine a set of future control moves in order to minimize a cost function based on a desired output trajectory over a prediction horizon. The computation involved in solving the optimization problem at every sampling time can become so intensive, particularly for high-dimensional systems, that it could make on-line applications almost impossible [1].

A fuzzy state observer is used to reconstruct, at least partially the state variables of the process. Different applications of state observers in bioprocess are reported in the literature [2, 3].

Model predictive control strategy

The MPC scheme is based on an explicit use of a prediction model of the system response to obtain control actions by minimizing a cost function. The optimization objectives include minimization of the difference between the predicted and desired response and the control effort subject to prescribed constraints such as limits on the magnitude of the control force. In the MPC scheme, first a reference response trajectory $r(k)$ is specified. The reference trajectory is the desired target trajectory of the structural response. This is followed by an appropriate prediction model which is then used to estimate the future building response $y(k)$. The prediction is made over a pre-established extended time horizon using the current time as the prediction origin. For a discrete time model, this means predicting for j sample times in the future. This prediction is based on the past control inputs $u(k)$, $u(k+1),\dots,$ $u(k+j)$ and on the sequence of future control efforts determined using the prediction model that are needed to satisfy a prescribed optimization objective. The control signals that were determined using the prediction model are then applied to the structure, and the actual structural system output $y(k)$ is found. Finally, the actual measurement $y(k)$ is compared to the model prediction and the prediction error is utilized to update future predictions. Fig. 1 describes schematically the basic MPC strategy.

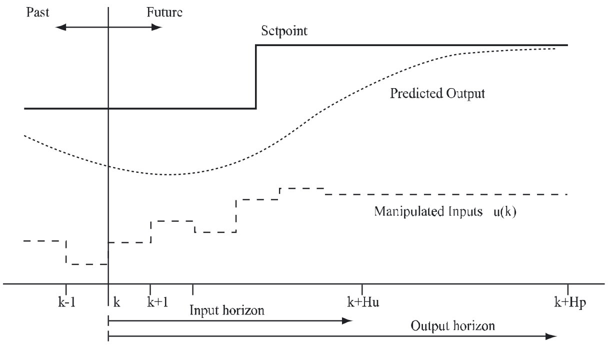


Fig. 1. Model predictive control strategy

Control system structure with fuzzy observer

First stage of predictive control strategy is to obtain an accurate model of the controlled system. In this paper a fuzzy-neural model is used as a predictive model in Nonlinear Model-Based Predictive Control (NMBPC) basic structure Fig. 2.

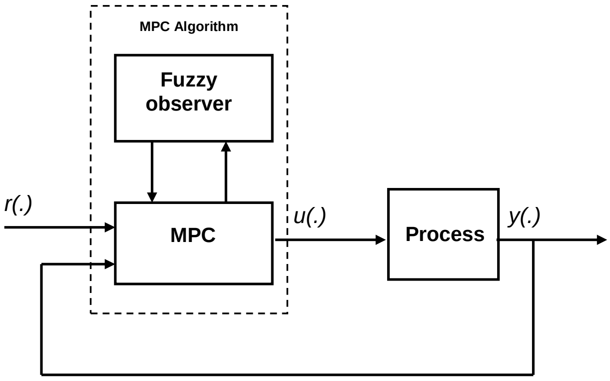


Fig. 2. Basic structure of NMBPC with Fuzzy Observer

The Takagi-Sugeno fuzzy-neural models are suitable for modelling a class of nonlinear systems [4]. As it is well known a wide class of nonlinear dynamic systems can be described in discrete time by a state-space model. The model in this paper is also taken in the state-space form:

$$\begin{aligned} x(k+1) &= f_x(x(k),u(k)) \\ y(k) &= f_y(x(k),u(k)), \end{aligned} \qquad (1)$$

where $x(k)$, $u(k)$ and $y(k)$ are vectors for the state, the input and the output of the plant respectively, with $dim(x(k))=n\times 1$, $dim(u(k))=m\times 1$, $dim(y(k))=q\times 1$. The unknown nonlinear functions f_x and f_y can be approximated by Takagi-Sugeno type fuzzy rules of the form [5,6]:

$$R_i:\ if \quad z_1(k)\ is\ M_{i1} \quad and \quad \dots \quad z_p(k)\ is\ M_{ip}\ , \\ then \quad \begin{cases} x_i(k+1) = A_i\, x(k) + B_i\, u(k) \\ y_i(k) = C_i\, x(k) \end{cases} \qquad (2)$$

where R_i is the i-th rule of the rule base, z_p are the state regressors (outputs and inputs of the system), M_{ip} is a membership function of a fuzzy set, A_i, B_i, C_i are state-space matrices with $dim(A_i)=n\times n$, $dim(B_i)=n\times m$, $dim(C_i)= q\times n$.

The state in the next sampling time and the output can be obtained taking the weighted sum of the activated rules, using

$$\begin{aligned} x(k+1) &= \sum_{i=1}^{L} \bar{\mu}_{yi}(k)(A_i\, x(k) + B_i\, u(k)) \\ \hat{y}(k) &= \sum_{i=1}^{L} \bar{\mu}_{yi}(k) C_i\, x(k) \end{aligned} \qquad (3)$$

On the other hand the state-space matrices A, B, C, and D for the global state-space plant model could be calculated as a weighted sum of the local matrices A_i, B_i, C_i, from the activated fuzzy rules:

$$A(k) = \sum_{i=1}^{L} A_i \bar{\mu}_{yi}(k) \qquad B(k) = \sum_{i=1}^{L} B_i \bar{\mu}_{yi}(k) \\ C(k) = \sum_{i=1}^{L} C_i \bar{\mu}_{yi}(k) \qquad (4)$$

where $\bar{\mu}_{yi} = \mu_{yi} \Big/ \sum_{i=1}^{L} \mu_{yi}$ is the normalized value of the membership function degree μ_{yi} upon the i-th activated fuzzy rule and L is the number of the activated rules. Fuzzy implication in the i-th rule (2) can be realized by means of a product composition.

$$\mu_{yi} = \mu_{1ij} \times \mu_{2ij} \times \dots \times \mu_{pij} \qquad (5)$$

where μ_{1ij}, μ_{2ij},...., μ_{pij} specify the membership degrees upon the activated j-th fuzzy set of the corresponded i-th input signal calculated according to the chosen Gaussian membership function:

$$\mu_{ij}(z_i) = \exp - \frac{(z_i - c_{Gij})^2}{2\sigma_{ij}^2} \qquad (6)$$

where z_i is the current input value of the i-th model input, c_{Gij} is the center (position) and σ_{ij} is the deviation (wide) of the *j*-th membership function.

In general, an observer designed for the model (3) has the form

$$\hat{x}(k+1)=\sum_{i=1}^{L}\overline{\mu}_{yi}(\hat{z}_i)(A_i\,\hat{x}(k)+B_i\,u(k)+G_i\,(y(k)-\hat{y}(k))$$
$$\hat{y}(k)=\sum_{i=1}^{L}\overline{\mu}_{yi}(\hat{z}_i)(C_i\,\hat{x}(k) \tag{7}$$

where z denotes the estimated scheduling vector and G_i, $i=1, \ldots, L$, are the observer gains. The observer design problem is to calculate the values of G_i, $i=1,\ldots,L$ such that the estimation error converges to zero. The estimation error can be written as

$$e(k)=x(k)-\hat{x}(k) \tag{8}$$

Substituting (3) and (7) into (8) yields

$$e(k)=\sum_{i=1}^{L}\overline{\mu}_{yi}(\hat{z}_i)(A_i\,\hat{x}(k)+B_i\,u(k))- -\sum_{i=1}^{L}\overline{\mu}_{yi}(\hat{z}_i)(A_i\,\hat{x}(k)+B_i\,u(k)+G_i\,(y(k)-\hat{y}(k)) \tag{9}$$

where $\hat{z}$ denotes the estimated vector of the state regressors.

Adding to and subtracting from the right-hand side of the above equation, after some algebraic manipulations we obtain

$$e(k+1)=\sum_{i=1}^{L}\overline{\mu}_{yi}(\hat{z}_i)(A_i\,e(k)-G_i\,(y(k)-\hat{y}(k))+ +\sum_{i=1}^{L}(\overline{\mu}_{yi}(z_i)-\overline{\mu}_{yi}(\hat{z}_i))(A_i\,\hat{x}(k)+B_i\,u(k)) \tag{10}$$

Since the output is measured, the membership functions of the measurement model do not depend on the estimated states. Therefore, we can rewrite (10) as

$$e(k+1)=\sum_{i=1}^{L}\left(\sum_{j=1}^{L}\overline{\mu}_{yi}(\hat{z}_i)\overline{\mu}_{yj}(z_i)(A_i-G_i\,C_i)e(k)\right)+ +\sum_{i=1}^{L}(\overline{\mu}_{yi}(z_i)-\overline{\mu}_{yi}(\hat{z}_i)(A_i\,\hat{x}(k)+B_i\,u(k)) \tag{11}$$

In order for the estimation error to converge to zero, the observer gains G_i have to be calculated such that the first term of (11) converges to zero and such that the disturbance due to the second term, $(\overline{\mu}_{yi}(z)-\overline{\mu}_{yi}(\hat{z}))$ becomes zero as $\hat{x}$ approaches x [7].

The observer gains G_i are usually computed using stability conditions developed for TS systems. The estimation error dynamics (11) is asymptotically stable, i.e., the estimation error converges to zero if there exists a positive definite matrix P such that:

$$F_{ij}PF_{ii}-P\le 0$$
$$\frac{(F_{ij}+F_{ji})^T}{2}P\frac{(F_{ij}+F_{ji})}{2}-P\le 0 \tag{12}$$

for all i, j such that $i < j$ and for x s.t. $\mu_i(x)\ \mu_j(x)\neq 0$ where $F_{ij}=A_i-G_iC_j$. The inequalities above can be transformed into a LMI problem. Next the values of G_i are substituted into the observer model.

A simplified fuzzy-neural identification approach is used for computation of the state-space matrices A_i, B_i, C_i, in (11) [5].

Predictive optimization algorithm

Second stage of the predictive control strategy includes optimization procedure, which uses the obtained in the first stage of modelling predictive model of the system. Using the Takagi-Sugeno fuzzy-neural model, the optimization algorithm computes the future control actions at each sampling period, by minimizing the typical cost function for MPC strategy:

$$J(k)=\sum_{i=Hw}^{H_p+H_w-1}\left\|\hat{y}(k+i)-r(k+i)\right\|^2 Q+\sum_{i=0}^{H_u-1}\left\|\Delta\hat{u}(k+i)\right\|^2 R, \tag{13}$$

where $\hat{y}(k)$, $r(k)$ and $\Delta\hat{u}(k)$ are vectors of predicted outputs, reference trajectories, and the predicted control increments at time k, respectively. $Q\geq 0$ and $R>0$ are weighting matrices representing the relative importance of each controlled and manipulated variable and they are assumed to be constant over the prediction horizon. The length of the prediction horizon is H_p, and the first sample to be included in the horizon is H_w, which may be used to compensate time delay. The control horizon is given by H_u.

The cost function (13) may be rewritten as

$$J(k)=\left\|\mathrm{Y}(k)-\mathrm{T}(k)\right\|^2\tilde{Q}+\left\|\Delta U(k)\right\|^2\tilde{R} \tag{14}$$

where $Y(k)=\begin{bmatrix}\hat{y}(k)\\ \vdots\\ \hat{y}(k+Hp-1)\end{bmatrix}, T(k)=\begin{bmatrix}r(k)\\ \vdots\\ r(k+Hp-1)\end{bmatrix},$

$$\Delta U(k)=\begin{bmatrix}\Delta\hat{u}(k)\\ \vdots\\ \Delta\hat{u}(k+Hu-1)\end{bmatrix}$$

$$\tilde{Q}=\begin{bmatrix} Q(1) & 0 & \cdots & 0 \\ 0 & Q(2) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & Q(H_p) \end{bmatrix} \quad \tilde{R}=\begin{bmatrix} R(1) & 0 & \cdots & 0 \\ 0 & R(2) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & R(H_u) \end{bmatrix}.$$

By deriving the prediction expressions, it can be written

$$Y(k) = \Psi x(k) + \Gamma u(k-1) + \Theta \Delta U(k) \tag{15}$$

where

$$\Psi = \begin{bmatrix} C \\ CA \\ CA^2 \\ \vdots \\ CA^{Hp-1} \end{bmatrix} \quad \Gamma = \begin{bmatrix} 0 \\ CB \\ CAB+CB \\ \vdots \\ C\sum_{i=0}^{Hp-2} A^i B \end{bmatrix}$$

$$\Theta = \begin{bmatrix} 0 & 0 & \cdots & 0 \\ CB & 0 & & \vdots \\ CAB+CB & & \ddots & 0 \\ \vdots & & \ddots & 0 \\ C\sum_{i=0}^{Hu-2} A^i B & & \cdots & 0 \\ \vdots & & \ddots & \vdots \\ C\sum_{i=0}^{Hp-2} A^i B & & \cdots & C\sum_{i=0}^{Hp-Hu-1} A^i B \end{bmatrix}.$$

The prediction error *E(k)* is defined as:

$$E(k) = T(k) - \Psi x(k) - \Gamma u(k-1) \tag{16}$$

This quantity could be interpreted as the free response of the system, if all variables at $t=k$, $\Delta U(k)$ were set to zero. Another formulation of the cost function (14)can be achieved by using the prediction expressions (15) and (16):

$$J(k) = \Delta U^T \mathrm{H}\, \Delta U - \Delta U^T \Phi + E^T Q E \tag{17}$$

where

$$\Phi = 2\Theta^T Q E(k), \quad H = \Theta^T Q \Theta + R \tag{18}$$

The problem of minimizing the cost function (18) is a quadratic programming problem. If *H* (Hessian matrix) is positive definite, the problem is convex [8]. The solution is given by closed form

$$\Delta \mathrm{U} = \frac{1}{2} H^{-1} \Phi \tag{19}$$

In general linear constraints, which take place in the quadratic programming problem, may be expressed in terms of $\Delta U(k)$ [5]. Therefore, the optimization problem can be rewritten as:

$$\begin{aligned} &\min\ J(k) = \Delta U^T H\ \Delta U - \Delta U^T \Phi + E^T Q E \\ &subject\ to\ \ \Omega \Delta U \le \omega \end{aligned} \tag{20}$$

where $\Omega \Delta \mathrm{U} \le \omega$ represents the constraints on outputs, inputs and variation of the inputs presented by $\Delta U(k)$.

The problem is still recognized as a QP problem, but in this formulation with linear inequality constraints (LICQP). Several of the most popular algorithms for constrained optimization are described in [8]. In the present paper the active-set method is used to solve the LICQP.

Description of the controlled evaporator

The scheme of the evaporator system considered in this paper as a controlled plant is shown on the Fig.3.

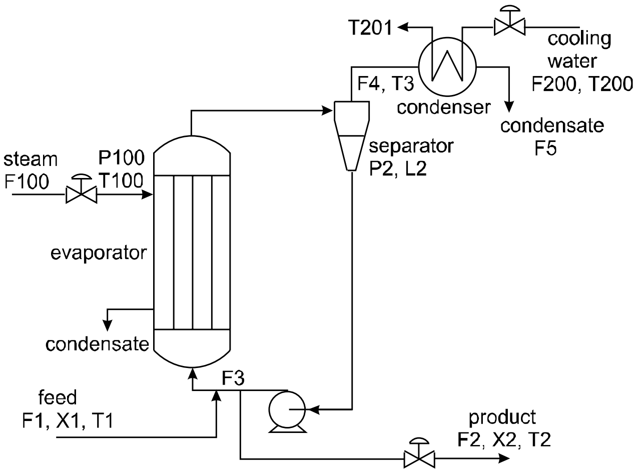


Fig. 3. Evaporator system

An evaporator system is an industrial process used to evaporate solvent from a feed-stream (e.g. in paper manufacture or sugar production).

The main variable which needs to be controlled is the 'Product Composition' X2, following the notation used in [1]. Keeping variations in the composition as small as possible maximizes the profitability of the evaporator, because it minimizes production of out-of-spec product (which cannot be sold, or has to be sold at a low price), while minimizing production costs, since it is possible to aim for a composition only a little better than the minimum acceptable one. However, it is also necessary to operate the evaporator safely, and without damaging the installed equipment. This requires the pressure in the evaporator (P2), and the level of liquid in the separator (L2), to be controlled. If the separator overflows the condenser will be damaged; if it runs dry the pump will be damaged. There are three variables which we assume we can adjust, in order to control the evaporator: the mass flow rate of the product being drawn off from the recirculating liquor (F2), the pressure of the steam entering the evaporator (P100), and the mass flow rate of the cooling water entering the condenser (F200). These three variables will be 'control inputs'.

There are several other variables which affect the evaporator's performance, but it is assumed that these are not under control. These will act as 'disturbances' which will keep pushing the controlled variables away from their target values. In particular, the Feed flow rate (F1), the circulating flow rate (F3), the Feed composition (X1), the Feed temperature (T1), and the cooling water inlet temperature (T200) all give the process significant disturbances.

Simulation experiments

Several experiments in Matlab/Simulink environment are performed in order to demonstrate how the proposed predictive algorithm works. The control system is configured with three manipulated variables u_1, u_2, u_3: F2, P100 and F200 and three controlled outputs y_1, y_2, y_3: L2, X2 and P2. The evaporation process has seven states [1]. Three of them are known and they much with the process outputs L2, X2 and P2 (Fig. 4). Unknown states of the process are modelled by the proposed fuzzy-neural observer. For the studied process they are considered to be P100, F3, F200, F2 (Fig. 3) after applying the first-order lag with a time constant equal to 0.5 min [1].

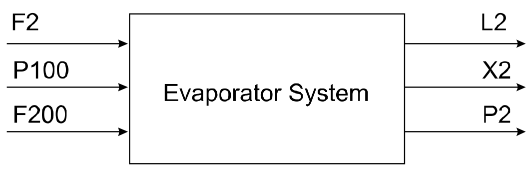


Fig. 4. Inputs and measured outputs of the evaporator system

The MPC tuning parameters used during the experiments are: prediction horizon H_p=30, control horizon H_u=15, weighting matrixes $Q=diag(100,0.1,0.1)$ and $R=diag(1,0.75,0.2)$. All manipulated variables and controlled outputs are constrained [9].

Results achieved during the experiment are presented in next figures. Fig. 5 shows the controlled outputs of the evaporation process. It could be seen that the measured outputs follow the changing with time setpoints quite well. The relevant control actions are depicted in Fig. 6.

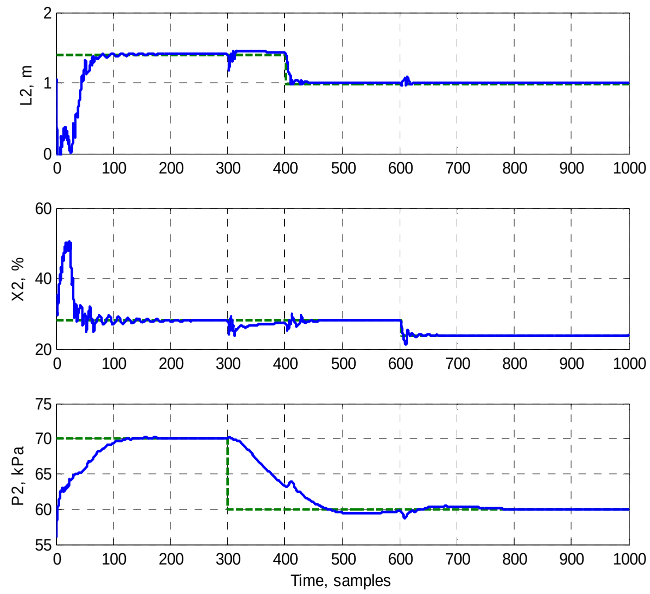


Fig. 5. Transient responses of the evaporator system

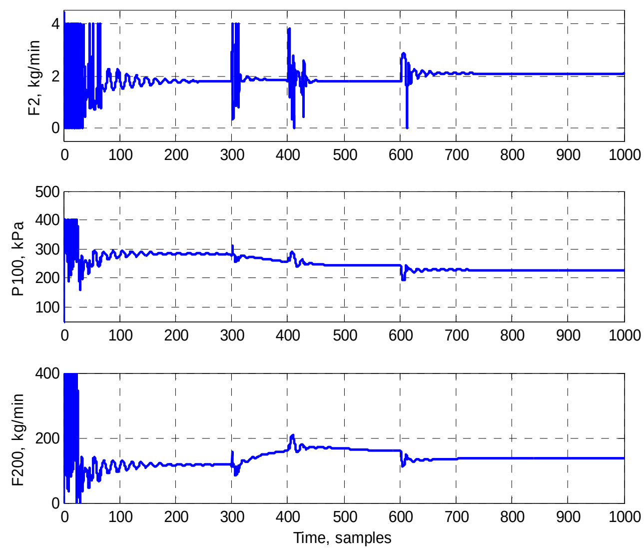
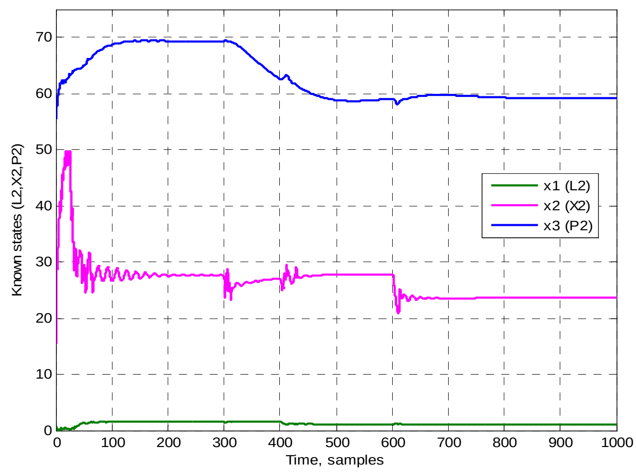


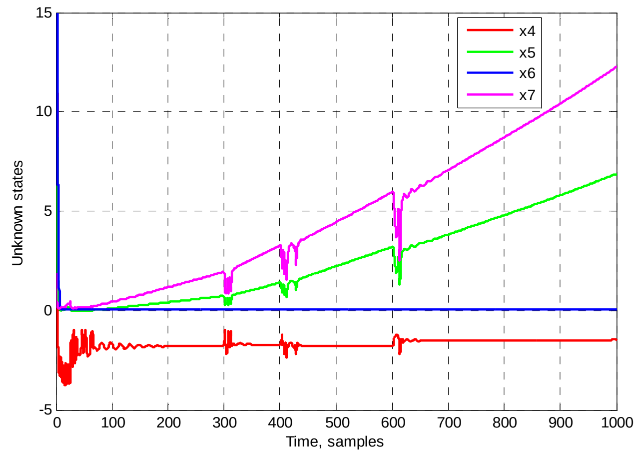
Fig. 6. Control actions applied to the evaporator system

Above figures shows that the satisfying results are achieved without violating the input constraints. The manipulated variables are constrained as follows $0 \leq F2 \leq 4$, $0 \leq P100 \leq 400$ and $0 \leq F200 \leq 400$.

Fig. 7 presents the results of applying the proposed in this paper fuzzy-neural model which includes an observer. All states of the studied process are achieved and successfully applied in the algorithm of MPC.



a)



b)

Fig. 7. Modelled states of the evaporation process a) known states b) estimated unknown states

Conclusions

In this paper, the model predictive control scheme was employed to reduce structural response of an evaporator system. Model predictive control was successfully applied to the studied evaporator system. The inherent instability of the system makes it difficult for modelling and control.

Adaptation of an observer is the most common way of dealing with plant's nonlinearities in practice. The results show that the controlled parameters have a good performance. The idea of using the fuzzy neural observer for nonlinear system identification is not new one, although more applications are necessary to show its capabilities in nonlinear identification and prediction. By implementing this idea to state-space representation of control systems, it is possible to achieve a powerful nonlinear model of plants or processes. Such models can be embedded into a model predictive control scheme. State-space model of the plant allows treating the optimization problem, as a quadratic programming task, which is an important part of MPC. It is important to note that the model predictive control approach has one major advantage that enables to include constraints of the system variables to the control algorithm.

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