Nonlinear Model Predictive Control of an Evaporator System Using Fuzzy-Neural Model

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Abstract: In this paper it is proposed a nonlinear approach to model predictive control that is based on a Takagi–Sugeno (TS) fuzzy model. An industrial evaporator system is taken as an exemplary process and its prediction model is used in the controller. Evaporators are widely used in the food processing industry to remove a portion of the water from food products. This reduces bulk and weight for subsequent processing, increases solids content (as for jams and molasses), helps preserve the product, provides convenience to the end consumer and concentrates color or flavor. Accurate nonlinear models of the evaporator system components are described. The final model of the evaporator system in state space implementation is used in model predictive control (MPC) scheme. Optimization objectives in MPC include minimization of the difference between the predicted and desired response trajectories, and the control effort subjected to prescribed constraints. The case study is implemented using MATLAB/Simulink. The simulation results show that the main process variables have good performance and process quality is satisfied. Copyright © 2012 IFAC

Keywords: Evaporator system, Model predictive control, Fuzzy-neural modelling, Process control.

1. INTRODUCTION

Model predictive control is a model-based control strategy that has been applied to a large number of industrial processes, where a sequence of future control actions is computed by minimizing an objective function. In the last two decades, linear model predictive control has been well recognized by industry due to its intuitiveness and capability to handle multivariable constraints. However, the extension to nonlinear model based predictive control (NMPC) has not been so successful although a significant amount of research effort has been put into this area. One of the main obstacles, which blocks NMPC techniques to become widely applicable, is the computational burden associated with the requirement to solve a set of nonlinear differential equations and a nonlinear dynamic optimization problem in real-time. The objective of NMPC is to determine a set of future control moves in order to minimize a cost function based on a desired output trajectory over a prediction horizon. The computation involved in solving the optimization problem at every sampling time can become so intensive, particularly for high-dimensional systems, that it could make on-line applications almost impossible (Maciejowski, 2002).

Different applications of fuzzy and fuzzy-neural structures for modeling of nonlinear systems are reported in the literature (Tanaka et al., 2001; Lee et al. 1994).

Case study of MPC of an evaporator system is implemented and examined using MATLAB/Simulink.

2. DESCRIPTION OF THE CONTROLLED EVAPORATOR SYSTEM

Many types of evaporators have certain common features to the one examined in this paper. The scheme of the evaporator system considered in this paper as a controlled plant is shown on the Fig.1.

Fig. 1. Evaporator system.

An evaporator system is an industrial process used to evaporate solvent from a feed-stream (e.g. in paper manufacture or sugar production).
A liquid ‘Feed’ comes in and is mixed with recirculating liquor. It is pumped into the evaporator itself, which is a heat exchanger, in which heat is exchanged with steam. Steam comes in at the top of the evaporator, and its condensate leaves at the bottom left. The recirculating liquor boils in the evaporator and a two-phase mixture of liquid and vapor flows to the Separator, where the liquid and vapor are separated from each other. The liquid, now more concentrated than when it entered the evaporator, is pumped round again, and some of it is drawn off as ‘Product’. The vapor flows to a Condenser, which is another heat exchanger, this time heat being exchanged with cooling water. The condensate, which may itself be a useful product in some cases, leaves the process, see the top right of Fig. 1.

The main variable which needs to be controlled is the ‘Product Composition’ $X_2$, following the notation used in (Newell, 1989). Keeping variations in the composition as small as possible maximizes the profitability of the evaporator, because it minimizes production of out-of-spec product (which cannot be sold, or has to be sold at a low price), while minimizing production costs, since it is possible to aim for a composition only a little better than the minimum acceptable one. However, it is also necessary to operate the evaporator safely, and without damaging the installed equipment. This requires the pressure in the evaporator ($P_2$), and the level of liquid in the separator ($L_2$), to be controlled. If the separator overflows the condenser will be damaged; if it runs dry the pump will be damaged.

There are three variables which we assume we can adjust, in order to control the evaporator: the mass flow rate of the product being drawn off from the recirculating liquor ($F_2$), the pressure of the steam entering the evaporator ($P_{100}$), and the mass flow rate of the cooling water entering the condenser ($F_{200}$). These three variables will be ‘control inputs’.

There are several other variables which affect the evaporator’s performance, but it is assumed that these are not under control. These will act as ‘disturbances’ which will keep pushing the controlled variables away from their target values. In particular, the Feed flow rate ($F_1$), the circulating flow rate ($F_3$), the Feed composition ($X_1$), the Feed temperature ($T_1$), and the cooling water inlet temperature ($T_{201}$) all give the process significant disturbances.

The Separator Level ($L_2$) is determined by equation

$$ L_2 = \int \frac{\rho A dL_2}{dt} = F_1 - F_2 - F_4 $$

where $\rho$ is the liquid density and $A$ is the cross-sectional area of the separator. It was assumed initially that $\rho A = 20$ kg/m$^3$.

The evaporator itself is modelled by five equations (Newell, 1989):

$$ \frac{dX_2}{dt} = F_1 \times X_1 - F_2 \times X_2 $$

where

$$ C \frac{dP_2}{dt} = F_4 - F_5 $$

$$ T_2 = 0.5616P_2 + 0.3126X_2 + 48.43 $$

$$ T_3 = 0.507P_2 + 55.0 $$

$$ F_4 = \frac{Q_{100} - F_1 \times C_p (T_2 - T_1)}{\lambda} $$

where $F_4$ is the vapor flow rate, $F_5$ is the condensate flow rate, $T_2$ and $T_3$ are the product and the vapor temperatures, respectively, $Q_{100}$ is the heater duty, and the coefficients have the following values: $M = 20$ kg, $C = 4$ kg/kPa, $C_p = 0.07$ kW/(kg/°K), $\lambda = 38.5$ kW/(kg/min).

Note that the signal $T_2$ will be needed as an output to the Steam Jacket model, $T_3$ will be needed as an output to the ‘Condenser’, and $F_4$ will be needed as an output to the ‘Separator’. Also note that, although $X_2$ and $P_2$ are not needed as inputs to other parts of the process, they are crucial outputs of the whole process, and will be among the most important variables being controlled. So it is crucial to bring them out as outputs of the evaporator.

The Heater Steam Jacket is described by equations (Newell, 1989):

$$ T_{100} = 0.1538 \times P_{100} + 90.0 $$

$$ Q_{100} = 0.16(F_1 + F_3)(T_{100} - T_2) $$

$$ F_{100} = \frac{Q_{100}}{\lambda_s} $$

where $T_{100}$ is the steam temperature, $F_{100}$ is the steam flow rate, and $\lambda_s$ is a coefficient with value $\lambda_s = 36.6$ kW/(kg/°K). Note that $Q_{100}$ is needed as an output to the evaporator.

The Condenser is described by equations (Newell, 1989):

$$ Q_{200} = \frac{UA_2(T_3 - T_{200})}{1 + \frac{UA_2}{2C_p F_{200}}} $$

$$ T_{201} = T_{200} + \frac{Q_{200}}{F_{200}} \times C_p $$

$$ F_5 = \frac{Q_{200}}{\lambda} $$

where $Q_{200}$ is the condenser duty, $T_{201}$ is the cooling water outlet temperature, and $UA_2$ is a coefficient with value $UA_2 = 6.84$ kW/°K. The other coefficients have the same values as above. Note that $F_5$ is needed as an output to the Evaporator.

Table 1 lists all the variables in the complete process model, and their nominal steady-state values.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>F1</td>
<td>Feed flowrate</td>
<td>10.0</td>
<td>kg/min</td>
</tr>
<tr>
<td>F2</td>
<td>Product flowrate</td>
<td>2.0</td>
<td>kg/min</td>
</tr>
<tr>
<td>F3</td>
<td>Circulating flowrate</td>
<td>50.0</td>
<td>kg/min</td>
</tr>
<tr>
<td>F4</td>
<td>Vapor flowrate</td>
<td>8.0</td>
<td>kg/min</td>
</tr>
<tr>
<td>F5</td>
<td>Condensate flowrate</td>
<td>8.0</td>
<td>kg/min</td>
</tr>
</tbody>
</table>
3. MODEL PREDICTIVE CONTROL SYSTEM

3.1 Model predictive control strategy

The MPC scheme is based on an explicit use of a prediction model of the system response to obtain control actions by minimizing a cost function. The optimization objectives include minimization of the difference between the predicted and desired response and the control effort subject to prescribed constraints such as limits on the magnitude of the control force. In the MPC scheme, first a reference response trajectory $r(k)$ is specified. The reference trajectory is the desired target trajectory of the structural response. This is followed by an appropriate prediction model which is then used to estimate the future building response $y(k)$. The prediction is made over a pre-established extended time horizon using the current time as the prediction origin. For a discrete time model, this means predicting $y(k)$ for $j$ sample times in the future. This prediction is based on the past control inputs $u(k), u(k+1), \ldots, u(k+j)$ and on the sequence of future control efforts determined using the prediction model that are needed to satisfy a prescribed optimization objective. The control signals that were determined using the prediction model are then applied to the structure, and the actual measurement $y(k)$ is compared to the model prediction and the prediction error is utilized to update future predictions.

Fig. 2 describes schematically the basic MPC strategy.

2.2 Control system structure with fuzzy-neural model

First stage of predictive control strategy is to obtain an accurate model of the controlled system. In this paper a fuzzy-neural model is used as a predictive model in Nonlinear Model-Based Predictive Control (NMBPC) basic structure (Fig. 2).

Fig. 3. Basic structure of NMBPC with fuzzy-neural model.

3.2 Takagi-Sugeno fuzzy modelling

The Takagi-Sugeno fuzzy-neural models are suitable for modelling a class of nonlinear systems. As it is well known a wide class of nonlinear dynamic systems can be described in discrete time by a state-space model. The model in this paper is also taken in the state-space form:

$$\begin{align*}
x(k+1) &= f(x(k), u(k)) \\
y(k) &= f(x(k), u(k))
\end{align*}
$$

where $x(k), u(k)$ and $y(k)$ are vectors for the state, the input, and the output of the plant respectively, $\text{dim}(x(k))=n \times 1$, $\text{dim}(u(k))=m \times 1$, $\text{dim}(y(k))=q \times 1$. The unknown nonlinear functions $f_x$ and $f_y$ can be approximated by Takagi-Sugeno type fuzzy rules of the form:

$$R_i: \text{if } z(k) \text{ is } M_{i1} \text{ and } \ldots \text{ z}(k) \text{ is } M_{iw}, \text{ then }
\begin{align*}
\dot{x}(k+1) &= A_i \dot{x}(k) + B_i u(k) \\
\dot{y}(k) &= C_i \dot{x}(k)
\end{align*}
$$

where $R_i$ is the $i$-th rule of the rule base, $z_p$ are the state regressors (outputs and inputs of the system), $M_{ip}$ is a membership function of a fuzzy set, $A_i, B_i, C_i$ are state-space matrices with $\text{dim}(A_i)=n \times n$, $\text{dim}(B_i)=n \times m$, $\text{dim}(C_i)=q \times n$.

The state in the next sampling time and the output can be obtained taking the weighted sum of the activated rules, using

$$\begin{align*}
\dot{x}(k+1) &= \sum_{i=1}^{i=p} \mu_{ni}(k)(A_i \dot{x}(k) + B_i u(k)) \\
\dot{y}(k) &= \sum_{i=1}^{i=p} \mu_{ni}(k)C_i \dot{x}(k)
\end{align*}
$$

On the other hand the state-space matrices $A, B, C,$ and $D$ for the global state-space plant model could be calculated as a weighted sum of the local matrices $A_i, B_i, C_i, D_i$ from the activated fuzzy rules:
where \( \mu_{ij}^k = \frac{\sum_{i=1}^{L} \mu_{ij}}{\sum_{i,j} \mu_{ij}} \) is the normalized value of the membership function degree \( \mu_{ij} \) upon the \( i \)-th activated fuzzy rule and \( L \) is the number of the activated rules. Fuzzy implication in the \( i \)-th rule (14) can be realized by means of a product composition.

\[
\mu_{ij} = \mu_{ij} \times \mu_{2ij} \times \cdots \times \mu_{p_{ij}}
\]

where \( \mu_{ij}, \mu_{2ij}, \ldots, \mu_{p_{ij}} \) specify the membership degrees upon the activated \( j \)-th fuzzy set of the corresponded \( i \)-th input signal calculated according to the chosen Gaussian membership function:

\[
\mu_{ij}(z_i) = \exp \left( \frac{-(z_i - c_{Gij})^2}{2 \sigma_{ij}^2} \right)
\]

where \( z_i \) is the current input value of the \( i \)-th model input, \( c_{Gij} \) is the center (position) and \( \sigma_{ij} \) is the deviation (wide) of the \( j \)-th membership function.

A simplified fuzzy-neural identification approach is used for computation of the space-state matrices \( A, B, C \), in (15) and (16). The identification procedure is realised by a five-layer fuzzy-neural network (Fig. 4).

**Fig. 4. Structure of the fuzzy-neural network applied for identification.**

The model obtained during the identification takes its part in the optimization procedure of MPC. Therefore, the importance of the accuracy of the models used in predictive control algorithms comes.

### 4. PREDICTIVE OPTIMIZATION ALGORITHM

Second stage of the predictive control strategy includes optimization procedure, which uses the obtained in the first stage of modelling predictive model of the system. Using the Takagi-Sugeno fuzzy-neural model, the optimization algorithm computes the future control actions at each sampling period, by minimizing the typical cost function for MPC strategy:

\[
J(k) = \sum_{i=1}^{L} \left[ \frac{1}{2} \| \hat{y}(k+i) - r(k+i) \|^2 Q + \sum_{i=0}^{H_u-1} \| \Delta u(k+i) \|^2 R \right],
\]

where \( \hat{y}(k), r(k) \) and \( \Delta u(k) \) are vectors of predicted outputs, reference trajectories, and the predicted control increments at time \( k \), respectively. \( Q \geq 0 \) and \( R > 0 \) are weighting matrices representing the relative importance of each controlled and manipulated variable and they are assumed to be constant over the prediction horizon. The length of the prediction horizon is \( H_p \), and the first sample to be included in the horizon is \( H_u \), which may be used to compensate time delay. The control horizon is given by \( H_u \).

The cost function (19) may be rewritten as

\[
J(k) = \left[ Y(k) - T(k) \right] Q^{-1} \left[ \hat{y}(k) - r(k) \right] + \left[ \Delta U(k) \right] R \left[ \hat{u}(k) - r(k) \right],
\]

where

\[
Y(k) = \left[ \begin{array}{c} y(k) \\ y(k+H_p-1) \end{array} \right], \quad T(k) = \left[ \begin{array}{c} r(k) \\ r(k+H_p-1) \end{array} \right], \quad \Delta U(k) = \left[ \begin{array}{c} \Delta u(k) \\ \Delta u(k+H_u-1) \end{array} \right]
\]

\[
\hat{y} = \left[ \begin{array}{c} Q(1) \\ 0 \\ \vdots \\ 0 \end{array} \right], \quad \hat{r} = \left[ \begin{array}{c} R(1) \\ 0 \\ \vdots \\ 0 \end{array} \right], \quad \hat{u} = \left[ \begin{array}{c} 0 \\ 0 \\ \vdots \\ 0 \end{array} \right]
\]

By deriving the prediction expressions, it can be written

\[
Y(k) = \Psi x(k) + \Gamma u(k - 1) + \Theta \Delta U(k),
\]

where

\[
\Psi = C A \left[ \begin{array}{c} 0 \\ C B \\ C A B + C B \\ \vdots \\ C A \end{array} \right], \quad \Gamma = \left[ \begin{array}{c} 0 \\ \vdots \\ 0 \\ C \end{array} \right], \quad \Theta = \left[ \begin{array}{c} 0 \\ \vdots \\ 0 \\ C \end{array} \right]
\]

The prediction error \( E(k) \) is defined as:

\[
E(k) = T(k) - \Psi x(k) - \Gamma u(k - 1).
\]

This quantity could be interpreted as the free response of the system, if all variables at \( t=k \), \( \Delta U(k) \) were set to zero. Another formulation of the cost function (20) can be achieved by using the prediction expressions (21) and (22):

\[
J(k) = \Delta U^T H \Delta U - \Delta U^T \Phi + \Delta U^T Q E
\]

where

\[
\Phi = 2 \Theta^T Q E(k), \quad H = \Theta^T Q \Theta + R
\]

The problem of minimizing the cost function (24) is a quadratic programming problem. If \( H \) (Hessian matrix) is positive definite, the problem is convex (Fletcher R., 2000). The solution is given by closed form
In general linear constraints, which take place in the quadratic programming problem, may be expressed in terms of $\Delta U(k)$ (Petrov et. al., 2011).

Therefore, the optimization problem can be rewritten as:

$$\min J(k) = \Delta U^T H \Delta U - \Delta U^T \Phi + E^T Q E$$

subject to $\Omega \Delta U \leq \omega$

where $\Omega \Delta U \leq \omega$ represents the constraints on outputs, inputs and variation of the inputs presented by $\Delta U(k)$.

The problem is still recognized as a QP problem, but in this formulation with linear inequality constraints (LICQP). Several of the most popular algorithms for constrained optimization are described in (Fletcher R., 2000). In the present paper the active-set method is used to solve the LICQP.

The algorithm of presented MPC using fuzzy-neural model is presented as a sequence of several steps (Table 2).

### Table 2. State-space implementation of fuzzy-neural Model Predictive Control strategy

<table>
<thead>
<tr>
<th>Step</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Read the current states, inputs and outputs of the system</td>
</tr>
<tr>
<td>2</td>
<td>Start identification of the fuzzy neural predictive model to determine the matrices $A(k)$, $B(k)$, $C(k)$</td>
</tr>
<tr>
<td>3</td>
<td>With $A(k)$, $B(k)$, $C(k)$ from Step 2 calculate the predicted output $Y(k)$ according to (21)</td>
</tr>
<tr>
<td>4</td>
<td>Obtain the prediction error $E(k)$ according to (22)</td>
</tr>
<tr>
<td>5</td>
<td>Construct the matrices of the cost function (23)</td>
</tr>
<tr>
<td>6</td>
<td>Solve the QP problem according to (26) taking into account the constraints</td>
</tr>
<tr>
<td>7</td>
<td>Calculate the control action $u(k) = u(k-1) + \Delta u(k)$</td>
</tr>
<tr>
<td>8</td>
<td>Apply only the first control action $u(k</td>
</tr>
</tbody>
</table>

5. SIMULATION EXPERIMENTS

Several experiments in MATLAB/Simulink environment are performed in order to demonstrate how the proposed algorithm works. The control system is configured with three manipulated variables $u_1$, $u_2$, $u_3$: $F_2$, $P_{100}$ and $F_{200}$ and three controlled values (outputs) $y_1$, $y_2$, $y_3$: $L_2$, $X_2$ and $P_2$ (Fig. 5). All manipulated variables are constrained as follows $0 \leq F_2 \leq 4$, $0 \leq P_{100} \leq 400$ and $0 \leq F_{200} \leq 400$.

Fig. 5. Inputs and outputs of the evaporator system.

Simulation results are shown in Fig. 6, 7, 8, 9. It can be seen from Fig. 6, 8 that measured outputs follow the setpoints quite well and this is achieved without violating the input constraints. The performance of the proposed fuzzy-neural MPC (FMPC) schema are compared to those of fuzzy PID (FPID) control introduced in (Taneva et. al., 2007), which is applied for each of the controlled output of the evaporator system. The MPC tuning parameters used during the experiments are set to be as follow: prediction horizon $H_p=30$, control horizon $H_u=7$, first sample to be included $H_w=3$, weighting matrices $Q=\text{diag}(100,0.1,0.2)$ and $R=\text{diag}(10,0.75,0.1)$.

During the first experiment the setpoints are constant during the experiment, and only measured disturbances, $F_1$, $X_1$, $T_1$, $F_3$ and $T_{200}$ are changed within ±20% of their nominal values after the 200th sample. The simulation experiment is performed with following initial values of the system outputs: $L=1$ m, $X_2 = 35.5$ %, $P_2 = 56.5$ kPa.

Fig. 6. Transient responses of the evaporator system with constant setpoints and changing disturbances.

Fig. 7. Control actions applied to the evaporator system with constant setpoints.
Next two figures (Fig. 8 and Fig. 9) show the results of the second experiment performed with changing reference. The initial values of the system outputs: \( L = 1 \) m, \( X_2 = 35.5 \% \), \( P_2 = 70 \) kPa. The disturbances \( F_1, X_1, T_1, F_3 \) and \( T_{200} \) have nominal values (Table 1).

![Fig. 8. Transient responses of the evaporator system with changing setpoints and constant disturbances.](image)

![Fig. 9. Control actions applied to the evaporator system with changing setpoints.](image)

The figures above present the results of the performed simulation experiments. It could be seen that the control inputs \( F_2, P_{100}, F_{200} \) of the evaporator system are close to their nominal values (Table 1) after achieving the corresponding setpoints (Fig. 7 and Fig. 9). Exception is the case where the measurable disturbances differ than their nominal values within ±20% (Fig. 6). It is obvious from Fig. 6 and Fig. 8. that the outputs are as close to desired values as the applied control actions are close to their nominal values. Hence, the aim of the applied control algorithms is to ensure these values.

Better performance of the applied FMPC could be seen on the figures. It is able to compensate the disturbances measured - \( F_1, X_1, T_1, F_3 \) and \( T_{200} \) (Fig. 6 and Fig. 7) and unmeasured – changing of setpoints (Fig. 8 and Fig. 9).

6. CONCLUSIONS

In this paper, the model predictive control scheme was employed to reduce structural response of an evaporator system. Model predictive control with fuzzy-neural model was successfully applied to the studied evaporator system. The inherent instability of the system makes it difficult for modelling and control.

The presented fuzzy-neural identification algorithm succeeds to provide an adequate and accurate model of the evaporator system even using only the measurable input/output information (without concerning its unmeasurable states). This reduce the computational burden without deteriorate the model accuracy and the control quality.

The state-space implementation of fuzzy-neural model to MPC strategy allows applying a multi-input multi-output controller to the examined evaporator system. The model uses the information for all inputs and outputs of the system to determine the control actions. This representation of Takagi-Sugeno fuzzy model is able to describe the full dynamics of the system. In contrast, the fuzzy PID algorithm relies only on the information for the output which it is applied for.

REFERENCES


