NONLINEAR MODEL BASED PREDICTIVE CONTROLLER USING A FUZZY-NEURAL WIENER-HAMMERSTEIN MODEL

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Abstract: It is presented in this paper a method for designing a nonlinear model predictive controller. The controller is based on a hybrid Wiener-Hammerstein fuzzy-neural predictive model and a simplified gradient optimization algorithm. The proposed approach is used to control the product temperature in a Lyophlization plant. The controller efficiency is tested and proved by simulation experiments in Matlab & Simulink.

Keywords: Model Predictive Control, Fuzzy models, Lyophlization

1 INTRODUCTION

The underlying philosophy of Model Predictive Control (MPC) consists of minimization of a performance objective function with respect to future input moves, over a finite time horizon. Simplicity of design combined with ability to tackle realities and interactions has helped MPC to achieve its current popularity in the process industry (Patwardahan *et al.* 1998).

In MPC a process dynamic model is used to predict future outputs over a prescribed period called prediction horizon. Afterward, the model outputs are used to compute the future control actions by minimizing the predefined cost function.

Most technological processes are inherently nonlinear. Under this circumstance, together with higher product quality specifications and increasing productivity demands require operating systems closer to the boundary of the admissible operating region. Linear models are often inadequate to describe the process dynamics and nonlinear models have to be used in these cases. Several excellent reviews of the main NMPC principles and advantages/disadvantages of can be found in (Rawlings *et al.* 2000), (Morari *et al.* (1999), (Findeisen *et al.* 2002).

Researchers have proposed several ways to equip MPC with the capability to deal with nonlinear processes. The approaches to Nonlinear Model Predictive Control (NMPC) can be categorized into two groups: those based on first principles models of the process and those based on black-box models identified from input-output data.

The nonlinear process models based on Artificial Neural Networks and Fuzzy Logic belong to the second group of black-box models. Typical for them is the precise description of the plant process by a set of linear submodels. In this way the design of a model predictive controller can be gratefully simplified. Also, with the fuzzy-neural models it is possible to cover a broad range of the process operating conditions.

One of the most frequently studied class of nonlinear models are the so-called block oriented nonlinear models, (Abonyi *et al.* 2000) which consist of the interconnection of Linear Time Invariant (LTI) systems and static nonlinearities. Within this class, two of the more common model structures are the Hammerstein and the Winner models.

The Hammerstein model consists of a cascade connection of static nonlinearity followed by a LTI system and its reverse representation lead to Wiener model structure.

Both models are successfully applied for nonlinear system representation in a number of practical approaches in the areas of chemical processes, biological processes (Gomez *et al.* 2003), signal processing (Pearson *et al.* 1995), communication and control (Stoica *et al.* 1981).

For instance, the nonlinear effects encountered in some industrial processes, such as distillation columns, pH-neutralization, heat exchangers, or electro-mechanical systems can be effectively modelled by a Hammerstein or a Wiener model.

It is presented in this paper a new approach for designing a hybrid Wiener-Hammerstein Fuzzy-Neural model (FNWH), based on so called "one step solutions" (Abonyi, *et al.* 2004). When the model is identified with the help of linguistic rules and data, gathered from the process, it has the potential to be transparent and easily interpretable (Babuska 1998). The proposed FNWH model is evaluated in a model based predictive control scheme.

Due to the relatively simple block-oriented structure, the application of Wiener-Hammerstein models in Model Predictive Control is more straightforward approach. In this paper, the FNWH model is implemented in MPC control scheme by using a simple fuzzy-neural approach and its efficiency is proved by simulation experiments in *Matlab* & *Simulink* environment to control the product temperature in a Lyophlization plant.

2 FUZZY-NEURAL WIENER-HAMMERSTEIN MODEL

The classical Wiener and Hammerstein models have the following structures (Fig.1):

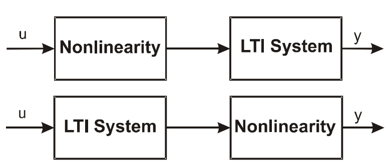

Fig.1. Structures of classical Hammerstein and Wiener models

When these structures are combined, both they give a new hybrid Wiener-Hammerstein model (Fig.2).

Fig.2. Structure of a hybrid Wiener-Hammerstein model

Using a simple fuzzy-neural approach the static nonlinearity can be easily approximated as a set of linear functions. For this purpose it is used the Takagi-Sugeno fuzzy-neural technique. As it is well known the Takagi-Sugeno fuzzy-neural technique is suitable to model a class of nonlinear dynamic systems, which can be described in discrete time by the NARX (Nonlinear AutoregRessive model with eXogenous inputs) input-output model. The used model for nonlinearity approximation is also taken in the NARX type:

$$s(k) = f_v(x(k)) \tag{1}$$

where the elements of the considered regression vector $x(k)$ are given by:

$$x(k) = [v(k),.........,v(k-n_v)] \tag{2}$$

$$v(k) = \frac{B(q^{-1})}{A(q^{-1})}u(k) \tag{3}$$

where

$$A(q^{-1}) = 1 + a_1q^{-1} + a_2q^{-2} + ... + a_mq^m$$
$$B(q^{-1}) = b_o + b_1q^{-1} + b_2q^{-2} + ... + b_nq^n$$

The unknown nonlinear functions f_v can be approximated by Takagi-Sugeno type fuzzy rules:

$$\begin{aligned} &R^{(i)} : \textit{if}\ \ \mathrm{x}_1 \text{ is } \tilde{\mathrm{A}}_1{}^{(i)} \text{ and } \mathrm{x}_\mathrm{p} \text{ is } \tilde{\mathrm{A}}_\mathrm{p}{}^{(i)} \\ &\text{then } \mathrm{f}_\mathrm{v}{}^{(i)}(k) \end{aligned} \tag{4}$$

$$\begin{aligned} &f_v{}^{(i)}(k) = r_1^{(i)}v(k) + r_2^{(i)}v(k-1) + ... \\ &+ r_{nv}^{(i)}v(k-n_v) + r_0^{(i)} \end{aligned} \tag{5}$$

(i)=1,2,…N, where *N* is the number of the fuzzy rules, A_i is an activated fuzzy set defined in the universe of discourse of the input $x=[x_1, x_2,....x_p]$ and the crisp coefficients *r* are the coefficients into the Sugeno function f_v. From a given input vector, the output of the fuzzy model is inferred by computing the following equation:

$$s(k) = f_v{}^{(i)}(k)g_v{}^{(i)} \quad \text{where} \quad g_v{}^{(i)} = \prod_{i=1}^{N}\mu_{vi}{}^{(i)} \tag{6}$$

where μ_{vi} are the degrees of fulfilment in notion to vi^{-th} activated fuzzy membership function.

Afterwards, the second linear part is introduced into the fuzzy model as follows:

$$\begin{aligned} &f_{v1}{}^{(i)}(k) = c_1{}^{(i)}y(k-1) + ...+ c_{n_y}{}^{(i)}y(k-n_y) + \\ &d_1{}^{(i)}s(k) + ...+ d_{n_v}{}^{(i)}s(k-n_v) + d_o \end{aligned} \tag{7}$$

Finally the output of the designed Fuzzy-Neural Hammerstein model is computed as:

$$\begin{aligned} y(k+1) &= \sum_{j=1}^{N}\left(\sum_{i=1}^{n_a}c_i y(k-i+1) + \sum_{i=1}^{n_b}d_i s_j(k-i-n_d+1)\right) \\ &= \sum_{j=1}^{N}\left(\sum_{i=1}^{n_a}c_i y(k-i+1) + \sum_{i=1}^{n_b}d_i r_j g_{vj}(v(k-i-n_d+1))\right) \end{aligned} \tag{8}$$

The designed FNWH model has the structure, as it shown on Fig. 3:

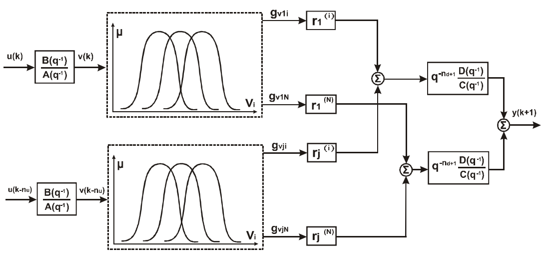

Fig.3. Fuzzy-Neural Wiener-Hammerstein model

In the FNWH model it is needed to be determined the unknown parameters – the number of membership functions, their shape, the parameters of the function f_v in the consequent part of the rules and the parameters into the linear parts. This is an identification procedure for which have been proposed numerous approaches. In this work it is applied a simplified fuzzy-neural approach, with a learning procedure described bellow.

2.1 Learning algorithm for the designed fuzzy-neural Wiener-Hammerstein model

It is used two steps simplified gradient learning procedure as a learning algorithm. This procedure is based on minimization of the instant error between the process output and the model output. It is needed to be adjusted two groups of parameters in the fuzzy-neural architecture – premise and consequent parameters. The consequent parameters are the coefficients r in the Sugeno function f_v and they are calculated by the following equations:

$$\beta_{ij}(k+1) = \beta_{ij}(k) + \eta(y(k) - y_M(k))\bar{g}_v^{(j)}(k)x_i(k) \tag{9}$$

$$\beta_{0j}(k+1) = \beta_{0j}(k) + \eta(y(k) - y_M(k))\bar{g}_v^{(j)}(k) \tag{10}$$

in which η is the learning rate and β_{ij} is an adjustable i^{-th} coefficient r_i in the Sugeno function f_v of the j^{-th} activated rule.

The premise parameters are the centre c_{ij} and the deviation σ_{ij} of an activated Gaussian fuzzy set. They can be calculated using the following equations:

$$c_{ij}(k+1)=c_{ij}(k)+\eta(y(k)-y_M(k))\bar{g}_v^{(j)}(k)[f_y^{(i)}(k)-\bar{y}(k)]\frac{[x_i(k)-c_{ij}(k)]}{c_{ij}^2(k)} \tag{11}$$

$$\sigma_{ij}(k+1)=\sigma_{ij}(k)+\eta(y(k)-y_M(k))\bar{g}_v^{(j)}(k)[f_y^{(i)}(k)-\bar{y}(k)]\frac{[x_i(k)-\sigma_{ij}(k)]^2}{\sigma_{ij}^3(k)} \tag{12}$$

To adjust the coefficients into the linear parts of the proposed FNWH model it is used the same gradient learning procedure as the described one above.

3 BASICS OF MODEL PREDICTIVE CONTROL STRATEGY

Predictive control is a general methodology for solving control problems in the time domain having one common feature: the controller is based on the prediction of the future system behaviour by using a process model. MPC is based on the use of an available model to predict the process outputs at future discrete times over a prediction horizon. A sequence of future control actions is computed using this model by minimizing a certain objective function. The good system performance depends on model accuracy and parameters in the objective function. Usually the receding horizon principle is applied, i.e., at each sampling instant the optimization process is repeated with new measurements, and the first control actions obtained are applied to the process (Camacho *et al.* 2004)

Nonlinear Model Predictive Control (NMPC) as it was applied with the Fuzzy-Neural Wiener-Hammerstein process model can be described in general with a block diagram, as it is depicted in Figure 4.

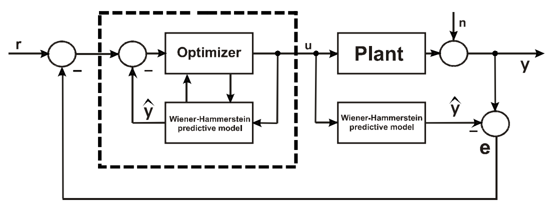

Fig.4. Block diagram of the proposed model predictive control system

Using the FN Wiener-Hammerstein model, the Optimization Algorithm computes the future control actions at each sampling period, by minimizing the following cost function:

$$J(k,v(k)) = \sum_{i=N_1}^{N_2}(r_e(k+i) - \hat{y}(k+i))^2 + \rho\sum_{i=1}^{N_u}\Delta v(k+i-1)^2 \tag{13}$$

where $\hat{y}$ is the predicted model output, r_e is the reference and v is the basic control action. The tuning parameters of the predictive controller are: N_1, N_2, N_u and ρ. N_1 is the minimum prediction horizon, N_2 is the maximum prediction horizon, N_u is the control horizon and ρ is the weighting factor penalizing changes in the control actions.

When the criterion function is a quadratic one and there are no constraints on the control action, as well the cost function can be minimized analytically. If the criterion J is minimized with respect to the future basic control actions v, then their optimal values can be calculated by applying the condition (Chitanov *et al.* 2003):

$$\nabla J[k,V(k)] = \left[\frac{\partial J[k,V(k)]}{\partial v(k)}, \frac{\partial J[k,V(k)]}{\partial v(k+1)}, \ldots, \frac{\partial J[k,vV(k)]}{\partial v(k+N_u-1)} \right] = 0 \tag{14}$$

$$\frac{\partial J[k,V(k)]}{\partial v(k)} = \left[-2[R_e(k) - \hat{Y}(k)]^T \frac{\partial \hat{Y}(k)}{\partial v(k)} + 2\rho \hat{V}(k)^T \frac{\partial \hat{V}(k)}{\partial v(k)} \right] \tag{15}$$

Finally, in order to obtain the proper value of the control action *u*, it is applied the reciprocal transformation from equation (3).

3.1 Optimization task calculation by using the Fuzzy-Neural Wiener-Hammerstein model

Since the FNWH model consists of a set of local linear models an explicit analytic solution of the above optimization problem can be obtained. Here is proposed a simplified method for calculation the elements of (14) based on the FNWH model. Hence, according to f_{v1} function (6) the unknown elements in (14) can be evaluated as follow:

$$\frac{\partial \hat{y}(k)}{\partial v(k)} = \sum_{i=1}^{N} d_1^{(i)} \overline{g}_v^{(i)}(k) \tag{16}$$

$$\frac{\partial \hat{y}(k+N_2)}{\partial v(k)} = \sum_{i=1}^{N} \begin{bmatrix} c_1^{(i)} \dfrac{\partial \hat{y}(k+N_2-1)}{\partial v(k)} + \ldots \\ + c_{ny}^{(i)} \dfrac{\partial \hat{y}(k+N_2-2)}{\partial v(k)} \end{bmatrix} \overline{g}_v^{(i)}(k+N_2) \tag{17}$$

$$\frac{\partial \hat{y}(k+N_1)}{\partial v(k)} = \sum_{i=1}^{N} \begin{bmatrix} c_1^{(i)} \dfrac{\partial \hat{y}(k+N_1-1)}{\partial v(k)} + \ldots. \\ + c_{ny}^{(i)} \dfrac{\partial \hat{y}(k+N_1-2)}{\partial v(k)} \end{bmatrix} \overline{g}_v^{(i)}(k+N_1) \tag{18}$$

4. EXPERIMENTAL RESULTS

4.1 Plant description

The assumed plant process in the following simulation experiments is a Lyophlization plant process.

Lyophlization process (Sublimation drying) is widely used in pharmaceutical and food industries, preparing stable dried medications and foodstuffs for astronauts and alpinists.

During the last years, extensive efforts by industry and research have been made to predict and optimize the course of the Lyophilization cycles in order to control the quality of the product and to minimize the costs (Shoen *et al.* 1995). According to this, the Model Predictive Control strategies are the natural choice to attend the mentioned requirements.

The main objective of the Lyophlization process is to remove the preliminary frozen water which takes part from the product structure by its sublimation. Using a Lyophlization plant the bound into the product water is removed by its transition from ice to vapor phase with no melting.

Referring to Fig.5 a simplified diagram of the main components of the Lyophilization plant is shown. The plant consists particularly of a drying chamber (1), temperature controlled shelves (2), a condenser (3) and a vacuum pump (4). The major purposes of the shelves are to cool and freeze or to supply heat to the product. This is supported by the shelves heater and refrigeration system (5). On those shelves the product is placed (6). The chamber is isolated from the condenser by the valve (7). The vacuum system is placed after condenser. After the process is completed the condenser will be heated in order to be removed the frozen ice from its wall.

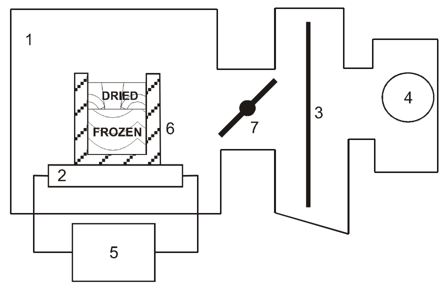

Fig.5. Schematic diagram of a simplified Lyophilization plant.

After the product is entirely frozen, the chamber is evacuated in order to increase the partial vapor water pressure difference between the frozen ice zone and the chamber. The shelf heating system starts to provide enthalpy for the sublimation process. The sublimation takes place at a moving ice front, which proceeds from the top of the frozen material downwards.

At the end of the primary drying, all the unconstrained water has been removed and what remains is the water which is constrained in the solution. At this point, the product can be removed, but in practice the water content is too high to guarantee biological stability. The stage in which the remaining water content is further reduced is called secondary drying, which takes place at higher temperature. In this contribution it is assumed only the first stage of the drying process called primary drying.

4.2 Sublimation in a vial

The participating ice is the unconstrained water accumulated in the small cylindrical tubes. Throughout the primary drying the product, which consists of the dried layer on the top and the frozen core at the bottom, stays below a certain temperature to insure that no melting occurs.

In this contribution it is assumed a small scale Lyophilization plant, for drying of 50 vials filled with glycine in water adjusted to pH 3, with hydrochloric acid.

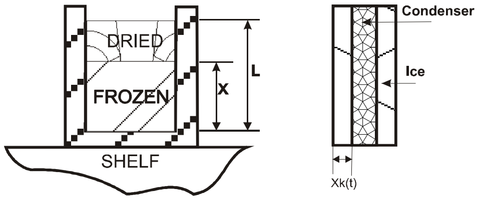

Fig.6. Schematic diagram of sublimation in a vial and condensation of the evaporated water at the condenser.

The schematic diagram on Fig.6. depicts the sublimation process occurring at the interface which is located at a distance *x* from the vial bottom. During sublimation the interface moves in a negative direction, while the product height remains constant. The sublimated water leaves product thought the already dried product layer.

The heat flow from the shelf to the vial consists of the heat conducted through the contact area between the vial and shelf. Since the vials have concave bottoms heat is also conducted through the enclosed gas between the vial and shelf. Energy inside the frozen product layer is lost due to the sublimated water and the conducted heat to the dried product layer. The heat flux from the chamber to the condensed depends on the thickness of the crystallized water at the condenser wall.

4.3 Initial conditions

There are made simulation experiments in *Matlab* environment to control the heating shelves temperature, in notion to temperature inside the frozen product layer. According to this circumstance, the system is nonlinear and non stationary one, and this is because during the sublimation process the properties of the product are changed.

The following initial conditions for simulation experiments are assumed:

- N_1=1, N_2=5, N_u=3,
- *System reference r=255 K*
- *Initial shelf temperature, before the start of the primary drying* $Ts_{in} = 228\ K$
- *Initial thickness of the interface front x=0.0023m*
- *Thickness of the product L=0.003m*

In the primary drying stage it is required to maintain the shelf temperature about 298 K, until the product will be dried. This circumstance requires about 45 minutes of time for the primary drying stage of the process.

4.4 Simulation experiments

There are made simulation experiments in *Matlab & Simulink* environment with the proposed FN Wiener-Hammerstein model with two different values of the weighting factor ρ. The used plant model for simulation in this study was derived from the physical laws of heat and mass transfer.

It has to be mentioned, that the reference point for the end of simulation experiments was taken the final value of the interface front at value *x=0.0001* m. This is due to the nature of the Lyophilization process.

As well, it can be mentioned also, that the main objective of the control system during the Lyophlization cycle is not to maintain the controlled temperature, in a classical sense. It is needed that the heat from the shelves to be provided with respect to drying regime requirements and constraints, in order to move the interface front in negative direction, as fast as possible. Afterward the shelves temperature is raised in order to remove the residual moist from the product, which in fact takes part from the secondary drying phase which is not considered in this contribution.

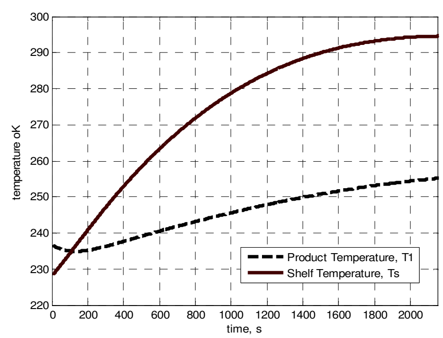

Fig.7. Product and Shelf temperatures in case of Model Predictive Control scheme with FNWH model at ρ=0.0003.

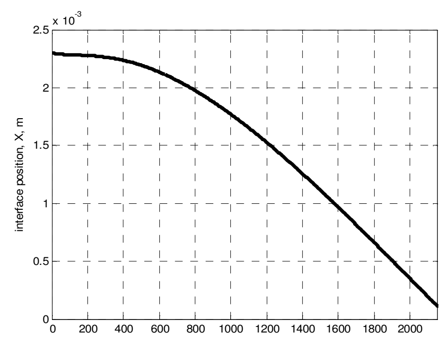

Fig.8. Interface position during the drying process in case of Model predictive control scheme with FNWH model at ρ=0.0003.

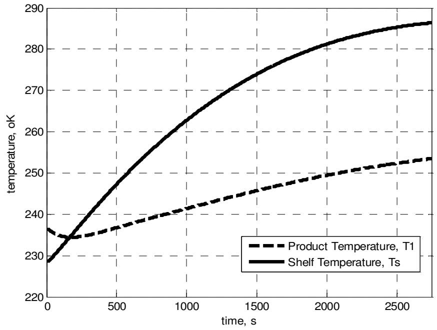

Fig.9. Product and Shelf temperatures in case of Model Predictive Control scheme with FNWH model at ρ=0.00022.

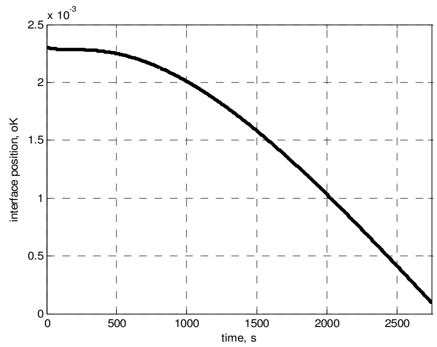

Fig.10. Interface position during the drying process in case of Model predictive control scheme with FNWH model at ρ=0.00022.

The temperature versus time profile for a representative vial is presented on Fig.7 and Fig.9. The primary drying phase for the cycle was started by increasing the shelf temperature from 228 K. The initial drop of the product temperature represents the sudden loss of heat due to sublimation and indicates the start of the primary drying. After, all of the unbound water has sublimated, the loss of heat due to sublimation vanishes and the enthalpy input from the shelf causes a sharp elevation of the product temperature.

The simulation results show the efficiency of the proposed MPC strategy based on a fuzzy-neural Wiener-Hammerstein model. Adjusting the parameter ρ causes intensification of the drying process and reducing of the drying time. The use of the proposed control strategy is also an effective energy saving solution for the process, since the maximum allowed shelf temperature is under its maximum bound.

5 CONCLUSIONS

It is presented in this paper a method for designing a nonlinear model predictive controller. The controller is based on a hybrid Wiener-Hammerstein fuzzy-neural predictive model and a simplified gradient optimization algorithm. The proposed approach was used to control the product temperature in a Lyophlization plant.

The simulation experiments show the efficiency of the proposed Model Predictive Control Strategy, based on the Fuzzy-Neural Wiener-Hammerstein model.

The product temperature in the frozen region rises according to Lyophilization cycle regime requirements and constraints. Adjusting the weighting factor penalizing changes of the control action, the transient time of the drying cycle can be minimized, which involves also, minimization of the additional energy cost into the system.

6 ACKNOWLEGMENTS

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