NMPC with Adaptive Learning Rate Scheduling of a Internal Fuzzy-Neural Model

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Abstract. It is presented in this paper a method for adaptive learning rate scheduling of the internal model in nonlinear model predictive controller. The controller is based on a Takagi-Sugeno fuzzy-neural model and a simplified gradient optimization algorithm. The proposed approach is used to control the level in a system with triple water tanks.

1 Introduction

Artificial Neural Networks (ANNs) are an efficient computational paradigm for solving many nonlinear problems. They have been applied successfully for identification and control of dynamic systems. One of the most attractive properties of the ANNs is their ability to learn on the base of input patterns. Learning means adjusting the weights so the input data are mapped onto minima of a quality or a cost function which is taken to be positive. For example backpropagation algorithm is such a learning procedure that adjusts the weights [1] by following rule:

$$w_{t+1} = w_t + \eta \nabla J(w_t) \tag{1}$$

through a steepest descent algorithm with respect to cost function J in weight space, where η is the so-called learning rate and w_t is a vector representing the weights at iteration step t. It is well known that if the learning rate is large, learning may occur quickly, but it may also become unstable. From the other hand, with a small learning rate the weights may adapt reliably, but it may take a long time and thus, it can invalidate the purpose of real-time operation. A class of asymptotically stable algorithms for learning rate adaptation can be found in [2]. Another problem in the neural networks learning is the algorithm convergence which is discussed in [3], [4], [5] and [6]. In [7] the authors are made an investigation in which situation is suitable to use a concrete learning rate adaptive method. In [11], [12], [13] and [14] it can be also found a novel advanced learning rate algorithms. In this paper it is proposed a method

for adaptive learning rate scheduling of the internal model in Generalized Predictive Controller. Generalized Predictive Control (GPC) belongs to the class of Model-Based Predictive Control (MPC) techniques and was firstly introduced by Clarke and his co-workers [9], [10]. In the predictive control scheme the model is used to predict the future behavior of the system. The accuracy of the model prediction directly determines the quality and effectiveness of the control law and is the primary consideration during implementation. Recently, several researchers have developed nonlinear model predictive control (NMPC) algorithms that work with different types of nonlinear models. These models can be divided into two classes. The first one includes models based on fundamental relationships. Such models can be accurate over a wide range of operating conditions, but they are very difficult to develop for many industrial cases. In addition, these models require tremendous computational effort for optimization and make them unsuitable for on-line applications. The second class models are based on empirical data, such as artificial neural networks and fuzzy logic models. Typical for this group is a precise description of the process by a set of linear submodels. In this way the design of a model predictive controller can be greatly simplified.
In the proposed NMPC with adaptive learning rate scheduling it is used the Takagi-Sugeno fuzzy-neural model, which is a quasi-linear empirical model. The predictive control scheme is modified by additional Learning Rate Scheduler for adaptive tuning of the internal model learning rate. The proposed approach is studied by experimental simulations in *Matlab* environment to control the level of a three cascaded water tanks.

2. Basics of model predictive control strategy

Model predictive control (MPC) is a common name for computer control algorithms that use an explicit process model to predict the future plant response. According to this chosen period, also known as the prediction horizon, the MPC algorithm optimizes the manipulated variable to obtain an optimal future plant response. The input of chosen length, also known as control horizon, is sent to the plant and then the entire sequence is repeated again in the next time period.
Nonlinear Model Predictive Control (NMPC) as it was applied with the Takagi-Sugeno fuzzy-neural process model can be described in general with a block diagram, as it is depicted in Fig. 1.

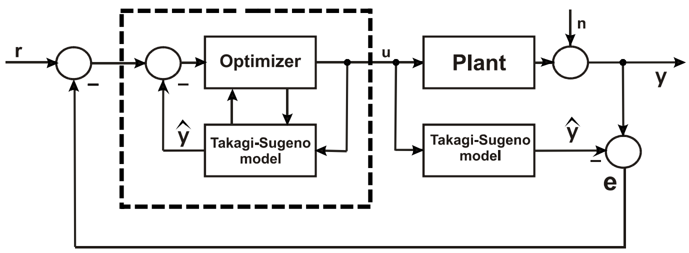

Fig. 1. Block diagram of model predictive control system

The Takagi-Sugeno fuzzy-neural models are suitable to model a class of nonlinear systems. As it is well known a wide class of nonlinear dynamic systems can be described in discrete time by the NARX (Nonlinear AutoregRessive model with eXogenous inputs) input-output model. The used model in this paper is also taken in the NARX type:

$$y(k)=f_y(x(k)) \tag{2}$$

where the unknown nonlinear function f_y can be approximated by Takagi-Sugeno type fuzzy rules:

$$R^{(i)}: if\ x_1\ is\ \widetilde{A}_1^{(i)}\ and\ x_p\ is\ \widetilde{A}_p^{(i)}\ then\ f_y^{(i)}(k) \tag{3}$$

$$f_y^{(i)}(k)=a_1^{(i)}y(k-1)+a_2^{(i)}y(k-2)+\cdots+a_{ny}^{(i)}y(k-n_y)+b_1^{(i)}u(k)+b_2^{(i)}u(k-1)+\cdots+b_{nu}^{(i)}u(k-n_u)+c_0^{(i)} \tag{4}$$

(i)=1,2,...N, where N is the number of the fuzzy rules, A_i is an activated fuzzy set defined in the universe of discourse of the input x_i and the crisp coefficients $a_1, a_2,...a_{ny}, b_1, b_2,...b_{nu}$ are the coefficients into the Sugeno function f_y.
In the Takagi-Sugeno fuzzy model is need to be determined the unknown parameters – the number of membership functions, their shape and the parameters of the function f_y in the consequent part of the rules. This is an identification procedure for which have been proposed numerous approaches. In this work is applied a simplified fuzzy-neural approach.

2.1 Learning algorithm for the designed fuzzy neural model

It is used two steps gradient learning procedure [8] as a learning algorithm of the internal fuzzy neural model. This procedure is based on the minimization of the instant error between the process output and the model output. It is need to be adjusted two groups of parameters in the fuzzy neural architecture – premise and consequent parameters. The consequent parameters are the coefficients $a_1, a_2,...a_n, b_1, b_2,...b_{nu}$ in the Sugeno function f_y and they are calculated firstly by following equations:

$$\beta_{ij}(k+1)=\beta_{ij}(k)+\eta(k)(y(k)-y_M(k))\overline{\mu}_y^{(j)}(k)x_i(k) \tag{5}$$

$$\beta_{0j}(k+1)=\beta_{0j}(k)+\eta(k)(y(k)-y_M(k))\overline{\mu}_y^{(j)}(k) \tag{6}$$

in which η is the learning rate and β_{ij} is an adjustable i^{-th} coefficient $(a_i$ or $b_i)$ in the Sugeno function f_y of the j^{-th} activated rule.
The premise parameters are the centre c_{ij} and the deviation σ_{ij} of a fuzzy set. They can be calculated using the following equations:

$$c_{ij}(k+1)=c_{ij}(k)+\eta(k)(y(k)-y_M(k))\overline{\mu}_y^{(j)}(k)[f_y^{(i)}(k)-\hat{y}(k)]\frac{[x_i(k)-c_{ij}(k)]}{c_{ij}^2(k)} \tag{7}$$

$$\sigma_{ij}(k+1)=\sigma_{ij}(k)+\eta(k)(y(k)-y_M(k))\overline{\mu}_y^{(j)}(k)[f_y^{(i)}(k)-\hat{y}(k)]\frac{[x_i(k)-\sigma_{ij}(k)]}{\sigma_{ij}^2(k)} \quad \textbf{(8)}$$

where c_{ij} and σ_{ij} are the centre and the deviation of a corresponding fuzzy set.
Using the Takagi-Sugeno fuzzy neural model, the Optimization Algorithm computes the future control actions at each sampling period, by minimizing the following cost function:

$$J(k,u(k))=\sum_{i=N_1}^{N_2}(r(k+i)-\hat{y}(k+i))^2+\rho\sum_{i=1}^{N_u}\Delta u(k+i-1)^2 \quad \textbf{(9)}$$

where $\hat{y}$ is the predicted model output, r is the reference and u is the control action. The tuning parameters of the predictive controller are: N_1, N_2, N_u and ρ. N_1 is the minimum prediction horizon, N_2 is the maximum prediction horizon, N_u is the control horizon and ρ is the weighting factor penalizing changes in the control actions.
When the criterion function is a quadratic one and there are no constraints on the control action, the cost function can be minimized analytically. If the criterion J is minimized with respect to the future control actions u, then their optimal values can be calculated by applying the condition:

$$\nabla J[k,U(k)]=\left[\frac{\partial J[k,U(k)]}{\partial u(k)},\frac{\partial J[k,U(k)]}{\partial u(k+1)},\ldots,\frac{\partial J[k,U(k)]}{\partial u(k+N_u-1)}\right]=0 \quad \textbf{(10)}$$

3. Adaptive learning rate scheduling by using a Takagi-Sugeno fuzzy-neural technique

As it was applied in classical model predictive control scheme for plant process approximation, the Takagi-Sugeno fuzzy-neural technique can be used also for design of adaptive learning rate scheduler based on the same principles. The designed model can be expressed in the following form:

$$\eta(k)=f_\eta(x(k)) \quad \textbf{(11)}$$

The unknown nonlinear function f_η can be approximated by Takagi-Sugeno type fuzzy rules:

$$P^{(i)}: if\ x_1\ is\ \widetilde{B}_1^{(i)}\ and\ x_s\ is\ \widetilde{B}_s^{(i)}\ then\ f_\eta^{(i)}(k) \quad \textbf{(12)}$$

$$f_\eta^{(i)}(k)=c_1^{(i)}\eta(k-1)+c_2^{(i)}\eta(k-2)+c_3^{(i)}e(k-1)+c_4^{(i)}e(k-2)+c_0^{(i)} \quad \textbf{(13)}$$

$$e(k)=y(k)-y_m(k) \quad \textbf{(14)}$$

(i)=1,2,…M, where M is the number of the fuzzy rules, η is the learning rate of the internal plant process model, e is the error between the actual process output y and estimated model output y_m, Bi is an activated fuzzy set defined in the universe of

discourse of the input x_i $x_i=[x_1, x_2 .. x_s]$ and the crisp coefficients, c_i, are the coefficients into the Sugeno function f_η. In the Takagi-Sugeno fuzzy model it is need to be determined the unknown parameters – the number of membership functions, their shape and the parameters of the function f_η in the consequent part of the rules, as well. As a learning procedure of the presented adaptive scheduler is considered the same two steps gradient algorithm discussed above (Equations (5)-(8)). The calculated output learning rate by the adaptive scheduler after defuzzyfication can be expressed as:

$$\eta(k) = \frac{\sum_{i=1}^{N} f_\eta^{(i)}(k) h_\eta^{(i)}}{\sum_{i=1}^{n} h_\eta^{(i)}} \quad where \quad h_\eta^{(i)} = \prod_{i=1}^{N} \mu_{\eta i}^{(i)} \tag{15}$$

It can be mentioned that the learning rate η estimated by the adaptive scheduler is common to each Takagi-Sugeno fuzzy-neural structure. On the other hand the described scheduler can be easily implemented as a neural network Fig. 2:

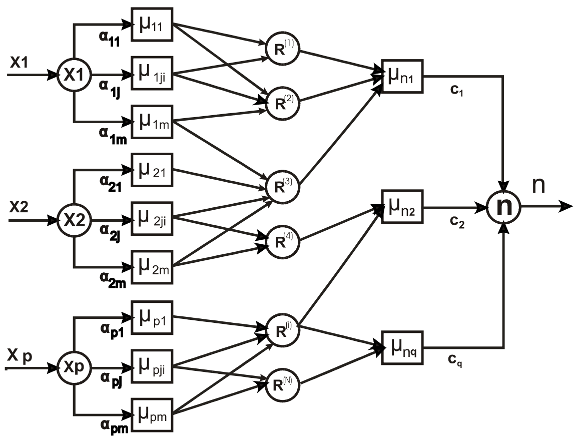

Fig. 2. The structure of the proposed ALrS neural network

Using the adaptive scheduler the classical model predictive control scheme will be modified as follow:

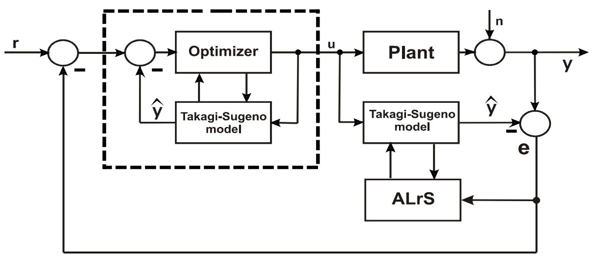

Fig. 3 Model predictive control scheme with adaptive learning rate scheduler

4. Simulation results

4.1 Plant description

The controlled process, considered in this work is a plant composed of three water tanks. It is presented at Figure 4. The main parameters of the process are: input flows Q_1 and Q_2; S – cross-sectional area of the tanks, equal for each one; S_a – outlet area of each tank; $h_{1,2,3}$ – the level in each tank.
The mathematical description of the plant is given bellow by equations (16), (17), (18). The aim of control task is maintaining the level of the third tank h_3 regarding the inflow Q_1 at constant inflow Q_2.

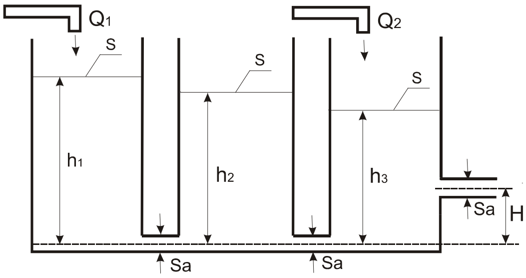

Fig. 4 The model of the three cascaded water tanks

$$\dot{h}_1 = -\frac{S_a}{S}F(h_1 - h_2) + \frac{1}{S}Q_1 \quad \textbf{(16)}$$

$$\dot{h}_2 = -\frac{S_a}{S}F(h_1 - h_2) - \frac{S_a}{S}F(h_2 - h_3) \quad \textbf{(17)}$$

$$\dot{h}_3 = -\frac{S_a}{S}F(h_2 - h_3) - \frac{S_a}{S}F(h_3 - H) + \frac{1}{S}Q_2 \quad \textbf{(18)}$$

4.2 Initial conditions for simulation

The simulation results are obtained with the next initial conditions:

- N_1=1, N_2=5, N_u=3
- Reference r=0.5 m.
- Variable reference: r=0.5m/0.6 m./0.55 m.

The chosen quality criteria in the control system are:

- System Overshoot - σ [%]
- Root Mean Squared Error

$$RMSE = \frac{1}{N}\sqrt{\sum_{k=1}^{N}(r(k)-y(k))^2} \tag{19}$$

4.3 Simulation experiments

At Fig.5 and Fig. 6 are shown transient process responses in cases of set and variable system reference. It is made comparison between the classical GPC control algorithm and the modified GPC algorithm with Adaptive Learning Rate Scheduling. The simulation results show the efficiency of the proposed modification in notion to improved system dynamics and quality control criterion - system overshoot which is reduced in case of using GPC algorithm with adaptive learning rate scheduling. A disadvantage of the proposed learning rate approach is the increased final value of the quality criterion RMSE.

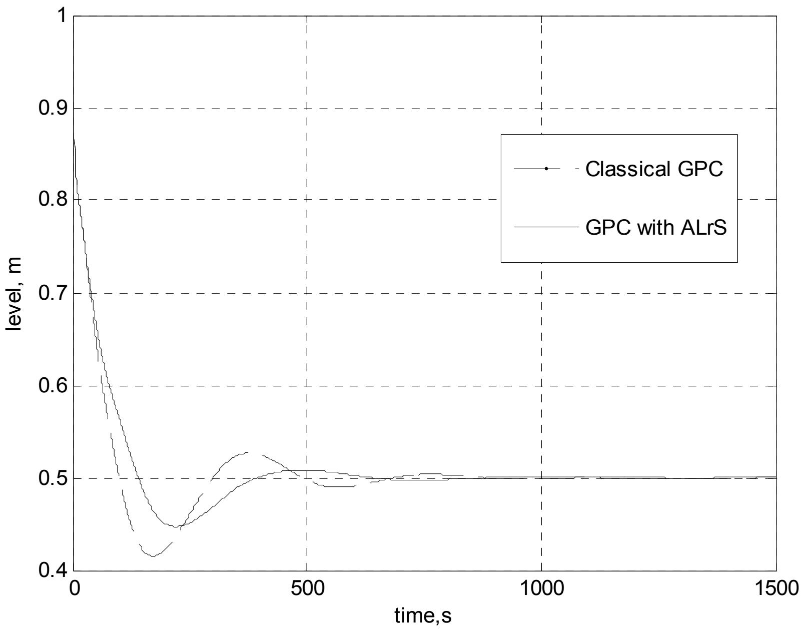

Fig. 5 Transient process responses in case of constant set point and ρ=0.001

Table 1. Quality control criteria estimation in case of variable system reference

Control algorithm	Reference			RMSE
	r =0.5m	r =0.6m	r =0.55m	
	σ [%]	σ [%]	σ [%]	
Classical GPC	33	60	40	0.04355
GPC with ALrS	21	33	25	0.07532

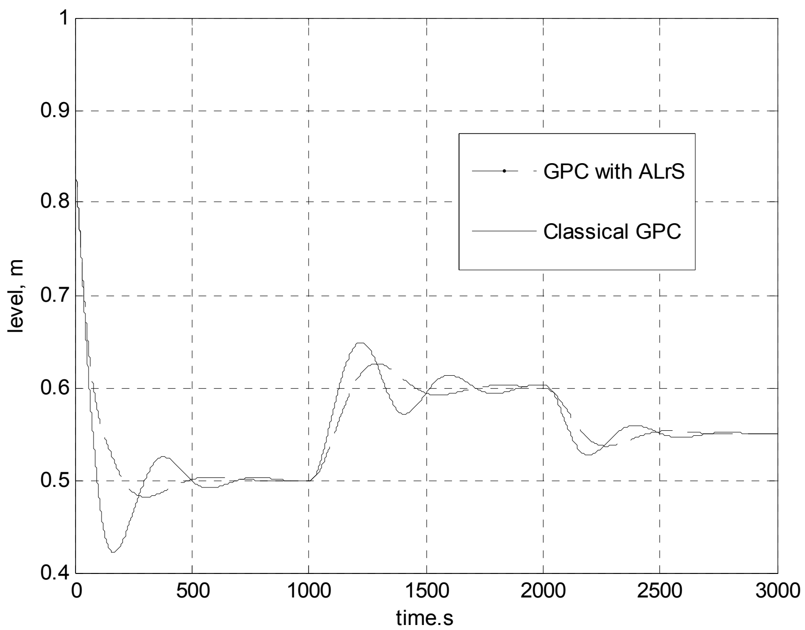

Fig. 6 Transient process responses in case of variable reference and ρ=0.0015

Table 2. Quality control criteria estimation in case of set system reference

Control algorithm	Reference r = 0.5 m	RMSE
	σ [%]	
Classical GPC	33	0.05673
GPC with ALrS	18	0.08138

5 Conclusions

A Takagi-Sugeno learning rate scheduler for adaptive learning of the internal model in classical predictive control scheme was presented for control of triple water tanks. It is made experimental simulations in Simulink environment with constant and variable reference signals. The simulation results show that in both cases the quality control criterion – system overshoot is significantly reduced but the final value of root mean squared error increases in case of using adaptive learning rate scheduler. The obtained results prove the efficiency of the proposed approach which leads to improved system dynamics.

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