Nonlinear Model Based Predictive Controller using a Fuzzy-Neural Hammerstein model

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Abstract. It is presented in this paper a method for designing a nonlinear model predictive controller. The controller is based on a Hammerstein fuzzy-neural predictive model and a simplified gradient optimization algorithm. The proposed approach is used to control the level in a system with triple water tanks.

1 Introduction

During the last decades, a considerable amount of research has been carried out one modeling, identification and control of nonlinear systems. Most of dynamical systems can be better represented by nonlinear models, which are able to describe the behavior of the system over the whole operating range, rather than by linear ones that are only able to approximate the system around a given operating point. One of the most frequently studied class of nonlinear models are the so-called block oriented nonlinear models [1], which consist the interconnection of Linear Time Invariant (LTI) systems and static nonlinearities. Within this class, two of the more common model structures are the Hammerstein and the Winner models.
The Hammerstein model consists a cascade connection of static nonlinearity followed by a LTI system [2] and it is successfully applied for nonlinear system representation in a number of practical approaches in the areas of chemical processes, biological processes, signal processing, communication and control [3], [4], [5]. For instance the nonlinear effects encountered in some industrial processes, such as distillation columns, pH-neutralization, heat exchangers, or electro-mechanical systems can be effectively modeled by a Hammerstein models.
Hammerstein models have also proven to be suitable for gray-box modeling where it is assumed that the steady-state behavior of the process is known a priori. The main disadvantage of this approach is that accurate first-principle steady-state models can rarely be obtained. Hence, black-box modeling techniques like neural networks and fuzzy logic can be used to approximate the nonlinearity.
Despite the simplified structure, the identification of Hammerstein models is a challenging task [5]. Several methods have been proposed in literature for

identification of Hammerstein models [5], [6], [7], [8], [9]. Depending on the data requirements and the type of implementation used, the available identification methods can be divided into the following three classes [7]:

Two-step approaches. The linear and nonlinear parts of the Hammerstein model are identified separately. If local dynamic data are available, the linear dynamic part is first identified and then the static nonlinearity is approximated. An example of this approach is given in [10]. If steady-state data are available, the first step is the identification of the static nonlinearity and the second one is the constrained identification of the unity-gain dynamic model. This approach has been used to integrate neural networks with linear dynamic models [11].

One-step, iterative solutions. This class includes techniques that alternately refine the estimate of the static nonlinearity and the dynamic linear model [6].

One-step, non-iterative solutions. The main feature in this approach is the use of predefined basis functions to approximate the nonlinearity by a model that is linear in its parameters. These parameters can be estimated by standard linear techniques.

This paper presents a method for designing a Fuzzy-Neural Hammerstein (FNH) model based on one-step solutions. When the model is identified with the help of linguistic rules and data, gathered from the process, it has the potential to be transparent and easily interpretable [12]. The proposed FNH model is evaluated in a model based predictive control scheme.

The concept of Model Predictive Control (MPC) has been heralded as one of the most significant control developments in recent years. The wide range of choice of model structures, prediction horizons and optimization criteria allows the designer to tailor the MPC to the process. In MPC a process dynamic model is used to predict future outputs over a prescribed period called prediction horizon. Afterward the model outputs are used to compute the future control actions by minimizing a cost function, over the prediction horizon. The good system performance depends on model accuracy and parameters in the objective function – weighting factors, prediction and control horizons. Some of the most applied predictive control algorithms in various industrial processes are: Dynamic Matrix Control (DMC) [13], Generalized Predictive Control (GPC) and Model Algorithmic Control (MAC) [14].

Due to the relatively simple block-oriented structure, the application of Hammerstein models in Model Predictive Control is more straightforward approach. In this paper, the FNH model is implemented in MPC control scheme by using a simple fuzzy-neural approach and its efficiency is proved by simulation experiments in Matlab & Simulink environment to control the level in a system with triple water tanks.

2 Fuzzy-Neural Hammerstein model

The classical Hammerstein model consists of a series connection of a static nonlinearity and linear time invariant dynamics (LTI) as it shown in Fig. 1.

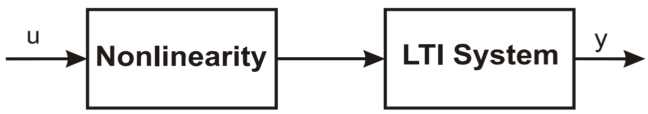

Fig. 1. Classical Hammerstein model

Using a simple fuzzy-neural approach the static nonlinearity can be easily approximated as a set of functions. For this purpose it is used the Takagi-Sugeno fuzzy-neural technique. As it is well known the Takagi-Sugeno fuzzy-neural technique is suitable to model a class of nonlinear dynamic systems, which can be described in discrete time by the NARX (Nonlinear AutoregRessive model with eXogenous inputs) input-output model. The used model for nonlinearity approximation is also taken in the NARX type:

$$u_m(k) = f_u(x(k)) \tag{1}$$

where the elements of the considered regression vector $x(k)$ are given by:

$$x(k) = [u(k), \ldots\ldots, u(k-n_u)] \tag{2}$$

The unknown nonlinear functions f_u can be approximated by Takagi-Sugeno type fuzzy rules:

$$R^{(i)} : if\ x_1\ is\ \tilde{A}_1^{(i)}\ and\ x_p\ is\ \tilde{A}_p^{(i)}\ then\ f_u^{(i)}(k) \tag{3}$$

$$f_u^{(i)}(k) = d_1^{(i)}u(k) + d_2^{(i)}u(k-1) + \ldots.. + d_{nu}^{(i)}u(k-n_u) + d_0^{(i)} \tag{4}$$

$(i)=1,2,\ldots N$, where N is the number of the fuzzy rules, A_i is an activated fuzzy set defined in the universe of discourse of the input $x=[x_1, x_2, \ldots x_p]$ and the crisp coefficients d_{ji}, are the coefficients into the Sugeno function f_u. From a given input vector, the output of the fuzzy model is inferred by computing the following equation:

$$u_m(k) = f_u^{(i)}(k) g_u^{(i)} \quad where \quad g_u^{(i)} = \prod_{i=1}^{N} \mu_{ui}^{(i)} \tag{5}$$

where μ_{ui} are the degrees of fulfillment in notion to ui-th activated fuzzy membership function.

Afterwards the linear part is introduced into the fuzzy model as follows:

$$f_s^{(i)}(k) = a_1^{(i)}y(k-1) + \ldots.. + a_{n_y}^{(i)}y(k-n_y) + b_1^{(i)}u_m(k) + \ldots\ldots + b_{n_u}^{(i)}u_m(k-n_u) + b_o \tag{6}$$

Finally the output of the designed Fuzzy-Neural Hammerstein model is computed as:

$$
\begin{aligned}
y(k+1) &= \sum_{j=1}^{N}\left(\sum_{i=1}^{n_a} a_i y(k-i+1) + \sum_{i=1}^{n_b} b_i u_{mj}(k-i-n_d+1)\right) \\
&= \sum_{j=1}^{N}\left(\sum_{i=1}^{n_a} a_i y(k-i+1) + \sum_{i=1}^{n_b} b_i d_j g_{uj}(u(k-i-n_d+1))\right)
\end{aligned}
\quad \textbf{(7)}
$$

The designed FNH model has the structure, as it shown at Fig. 2:

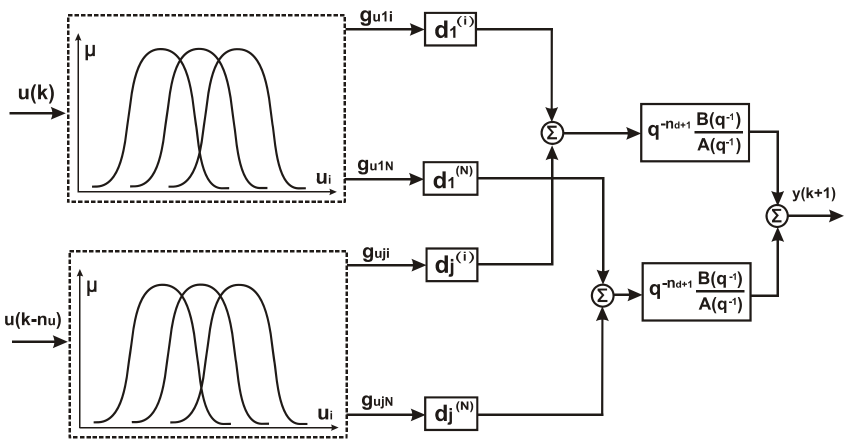

Fig. 2. Fuzzy-Neural Hammerstein model

In the FNH model is needed to be determined the unknown parameters – the number of membership functions, their shape and the parameters of the function f_u in the consequent part of the rules. This is an identification procedure for which have been proposed numerous approaches. In this work is applied a simplified fuzzy-neural approach, with learning procedure described bellow.

2.1 Learning algorithm for the designed fuzzy-neural model

It is used two steps gradient learning procedure [15] as a learning algorithm of the internal fuzzy neural model. This procedure is based on the minimization of the instant error between the process output and the model output. It is needed to be adjusted two groups of parameters in the fuzzy-neural architecture – premise and consequent parameters. The consequent parameters are the coefficients d_{ji} in the Sugeno function f_u and they are calculated by the following equations:

$$\beta_{ij}(k+1) = \beta_{ij}(k) + \eta(y(k) - y_M(k))\overline{g}_u^{(j)}(k)x_i(k) \quad \textbf{(8)}$$

$$\beta_{0j}(k+1)=\beta_{0j}(k)+\eta(y(k)-y_M(k))\overline{g}_u^{(j)}(k) \quad \textbf{(9)}$$

in which η is the learning rate and β_{ij} is an adjustable i^{-th} coefficient d_i in the Sugeno function f_u of the j^{-th} activated rule.

The premise parameters are the centre c_{ij} and the deviation σ_{ij} of a activated Gaussian fuzzy set. They can be calculated using the following equations:

$$c_{ij}(k+1)=c_{ij}(k)+\eta(y(k)-y_M(k))\overline{g}_u^{(j)}(k)[f_y^{(i)}(k)-\hat{y}(k)]\frac{[x_i(k)-c_{ij}(k)]}{c_{ij}^2(k)} \quad \textbf{(10)}$$

$$\sigma_{ij}(k+1)=\sigma_{ij}(k)+\eta(y(k)-y_M(k))\overline{g}_u^{(j)}(k)[f_y^{(i)}(k)-\hat{y}(k)]\frac{[x_i(k)-\sigma_{ij}(k)]}{\sigma_{ij}^2(k)} \quad \textbf{(11)}$$

To adjust the coefficients a_i, b_i into the linear part of the proposed FNH model is used the same gradient learning procedure described above (8), (9).

3 Basics of model predictive control strategy

Predictive control is a general methodology for solving control problems in the time domain having one common feature: the controller is based on the prediction of the future system behavior by using a process model. MPC is based on the use of an available model to predict the process outputs at future discrete times over a prediction horizon. A sequence of future control actions is computed using this model by minimizing a certain objective function. The good system performance depends on model accuracy and parameters in the objective function. Usually the receding horizon principle is applied, i.e., at each sampling instant the optimization process is repeated with new measurements, and the first control actions obtained are applied to the process [17].

Nonlinear Model Predictive Control (NMPC) as it was applied with the Fuzzy-Neural Hammerstein process model can be described in general with a block diagram, as it is depicted in Figure 3.

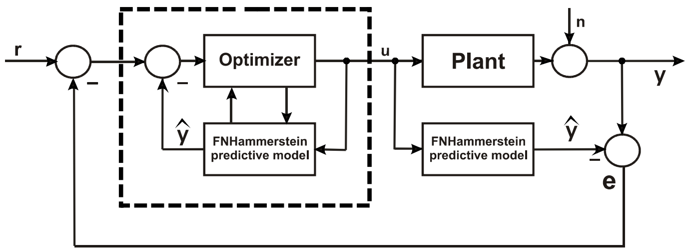

Fig. 3. Block diagram of model predictive control system

Using the FN-Hammerstein model, the Optimization Algorithm computes the future control actions at each sampling period, by minimizing the following cost function:

$$J(k,u(k)) = \sum_{i=N_1}^{N_2} (r(k+i) - \hat{y}(k+i))^2 + \rho \sum_{i=1}^{N_u} \Delta u(k+i-1)^2 \quad \textbf{(12)}$$

where $\hat{y}$ is the predicted model output, r is the reference and u is the control action. The tuning parameters of the predictive controller are: N_1, N_2, N_u and ρ. N_1 is the minimum prediction horizon, N_2 is the maximum prediction horizon, N_u is the control horizon and ρ is the weighting factor penalizing changes in the control actions.
When the criterion function is a quadratic one and there are no constraints on the control action, as well the cost function can be minimized analytically. If the criterion J is minimized with respect to the future control actions u, then their optimal values can be calculated by applying the condition [16]:

$$\nabla J[k,U(k)] = \left[\frac{\partial J[k,U(k)]}{\partial u(k)}, \frac{\partial J[k,U(k)]}{\partial u(k+1)}, \ldots, \frac{\partial J[k,U(k)]}{\partial u(k+N_u-1)} \right] = 0 \quad \textbf{(13)}$$

$$\frac{\partial J[k,U(k)]}{\partial U(k)} = \left[-2[R(k) - \hat{Y}(k)]^T \frac{\partial \hat{Y}(k)}{\partial U(k)} + 2\rho \hat{U}(k)^T \frac{\partial \hat{U}(k)}{\partial U(k)} \right] \quad \textbf{(14)}$$

3.1 Optimization task calculation by using the Fuzzy-Neural Hammerstein model

Since the FNH model consist a set of local linear models an explicit analytic solution of the above optimization problem can be obtained. Here is proposed a simplified method for calculation the elements of (14) based on the FNH model. Hence, according to f_s function (6) the unknown elements in (14) can be evaluated as follow:

$$\frac{\partial \hat{y}(k)}{\partial u(k)} = \sum_{i=1}^{N} b_1^{(i)} \overline{g}_u^{(i)}(k) \quad \textbf{(15)}$$

$$\frac{\partial \hat{y}(k+N_1)}{\partial u(k)} = \sum_{i=1}^{N} \left[a_1^{(i)} \frac{\partial \hat{y}(k+N_1-1)}{\partial u(k)} + \ldots. + a_{ny}^{(i)} \frac{\partial \hat{y}(k+N_1-2)}{\partial u(k)} \right] \overline{g}_u^{(i)}(k+N_1) \quad \textbf{(16)}$$

$$\frac{\partial \hat{y}(k+N_2)}{\partial u(k)} = \sum_{i=1}^{N} \left[a_1^{(i)} \frac{\partial \hat{y}(k+N_2-1)}{\partial u(k)} + \ldots + a_{ny}^{(i)} \frac{\partial \hat{y}(k+N_2-2)}{\partial u(k)} \right] \overline{g}_u^{(i)}(k+N_2) \quad \textbf{(17)}$$

4 Experimental results

4.1 Plant description

The controlled process, considered in this work is a plant composed of three water tanks. It is presented at Figure 4. The main parameters of the process are: input flows Q_1 and Q_2; S – cross-sectional area of the tanks, equal for each one; S_a – outlet area of each tank; $h_{1,2,3}$ – the level in each tank.
The mathematical description of the plant is given bellow by equations (18), (19), (20). The aim of control task is maintaining the level of the third tank h_3 regarding the inflow Q_1 at constant inflow Q_2.

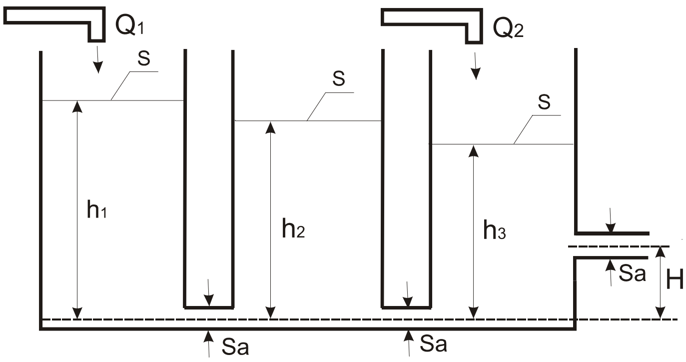

Fig. 4. The model of the three cascaded water tanks

$$\dot{h}_1 = -\frac{S_a}{S}F(h_1 - h_2) + \frac{1}{S}Q_1 \quad \textbf{(18)}$$

$$\dot{h}_2 = -\frac{S_a}{S}F(h_1 - h_2) - \frac{S_a}{S}F(h_2 - h_3) \quad \textbf{(19)}$$

$$\dot{h}_3 = -\frac{S_a}{S}F(h_2 - h_3) - \frac{S_a}{S}F(h_3 - H) + \frac{1}{S}Q_2 \quad \textbf{(20)}$$

4.2 Initial conditions for simulation

he simulation results are obtained with the next initial conditions:

- N_1=1, N_2=5, N_u=3
- Variable reference: r=0.5m/0.6 m./0.55 m.

The chosen quality criteria in the control system are:

- System Overshoot - σ [%]
- Root Mean Squared Error

$$RMSE = \frac{1}{N}\sqrt{\sum_{k=1}^{N}(r(k)-y(k))^2} \qquad (21)$$

4.3 Simulation results

It is made a simulation experiments in Matlab & Simulink environment with the proposed FN-Hammerstein model. There are considered two cases: of training the designed fuzzy-neural model with sinusoidal signal and control the level in a system with triple water tanks. The simulation results prove the effectiveness of the designed FN-Hammerstein model in notion to expected system behavior and quality control criteria.

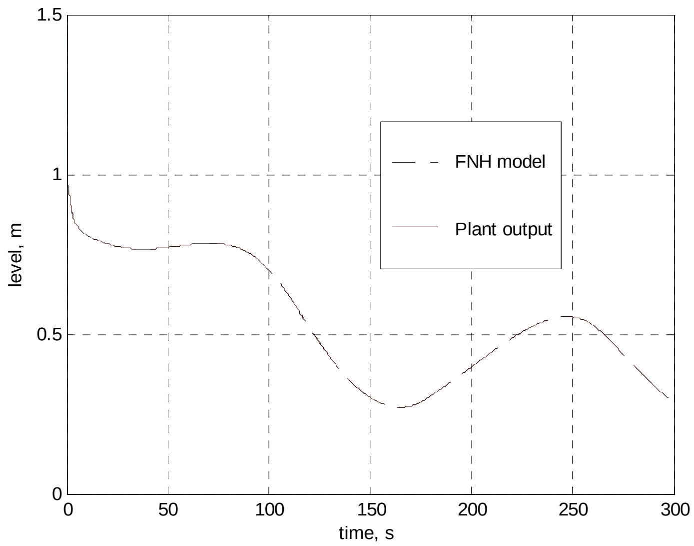

Fig. 5. Training of the FN-Hammerstein model with sinusoidal signal

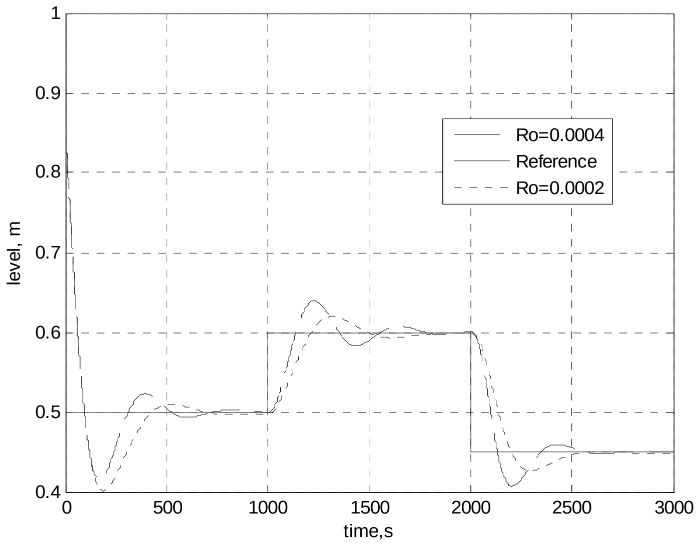

Fig. 6. GPC using Fuzzy-Neural Hammerstein model at different values of ρ

Table 1. Quality control criteria

ρ	r=0.5m.	r=0.6m.	r=0.55m.	RMSE
		σ [%]		
0.0002	15	25	11	0.003065
0.0004	21	34	25	0.002856

5 Conclusions

It is presented in this paper a method for designing a Fuzzy-Neural Hammerstein model based on a Takagi-Sugeno fuzzy-neural technique. The system nonlinearity is approximated by using a fuzzy-neural TS type model and the fuzzy rules are extended with the linear part of the model. The designed FNH model is used in a scheme of generalized predictive controller and the simulation results show its efficiency. The training of the designed FNH model shows the excellent tracking of the system behavior. The results in case of GPC control show the good system performance in case of variable system reference using different values of the weighting factor ρ.

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