# МНОГОМЕРЕН НЕЛИНЕЕН НЕВРОННО-РАЗМИТ ПРЕДСКАЗВАЩ РЕГУЛАТОР

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*Резюме: В настоящият доклад е представен алгоритъм на многомерен нелинеен предсказващ регулатор, базиран на невронно-размит модел Такаги-Сугено. За изчисляване на коефициентите в нелинейния невронно-размит модел и оптималните стойности на управлението е използван градиентен метод. При симулации, проведени в средата Matlab, предложеният нелинеен предсказващ алгоритъм е използван за регулиране нивото на система от каскадно свързани резервоари.*

# MULTIVARIABLE NONLINEAR FUZZY-NEURAL PREDICTIVE CONTROLLER

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*Abstract: In this paper is presented a method for designing a multivariable nonlinear predictive controller based on a Takagi-Sugeno fuzzy-neural model. It is used a simplified gradient technique in optimization task to calculate predictions on the future control actions and for on-line adaptation of the fuzzy-neural model. The proposed nonlinear predictive algorithm is used to control level in a system with quadruple water tanks.*

## 1. INTRODUCUTION

Model Predictive Control (MPC) has emerged as one of the most attractive control technique during the past decade. In MPC a process dynamic model is used to predict future outputs over a prescribed period called prediction horizon. Some of the most applied predictive control algorithms in various industrial processes are: Dynamic Matrix Control (DMC) [1], Generalised Predictive Control (GPC) and Model Algorithmic Control (MAC) [4].

In general, a wide range of industrial processes are inherently nonlinear. For such nonlinear systems, a linear MPC algorithm may not insure satisfactory dynamic performance. Recently, several researchers have developed nonlinear model predictive control (NMPC) algorithms that work with different types of nonlinear models. Some of these models are based on empirical data, such as artificial neural networks and fuzzy logic models.

The Takagi-Sugeno model is a quasi-linear empirical model developed by means of fuzzy logic for each local subsystem. The whole process behaviour is characterized by weighted sum of the outputs from all quasi-linear fuzzy implications. It is also well known that this modelling technique is applied successfully to a large class single-input, single–output (SISO), as well as multi-input, single-output (MISO) nonlinear dynamic processes. Recently, also methods have been proposed to deal with multi-input, multi-output (MIMO).

In the classical predictive control scheme the model outputs are used to compute the future control actions by minimizing a cost function, over the prediction horizon. The good system performance depends on model accuracy and parameters in the objective function.

It is presented in this paper a nonlinear predictive control strategy based on a MIMO Takagi-Sugeno fuzzy-neural model. The predictive control algorithm is modified to compute a multivariable optimization task. The proposed approach is studied by experimental simulations in *Matlab* environment to control the level of a quadruple water tanks.

## 2. MIMO TAKAGI-SUGENO MODELING

The Takagi-Sugeno fuzzy-neural models are suitable to model a class of nonlinear systems. The fuzzy-neural model, considered in this paper for constructing a MIMO process model, works with Gaussian membership functions for fuzzyfication, product inference engine in rule base and a weighted average defuzzifier. Simply the model can be described as:

$$y_j(k) = f_j(x(k)) \tag{1}$$

$$f_j^{(i)}(k) = \sum_{j=1}^{r}\sum_{l=1}^{n_y} a_{jl}^{(i)} y_j(k-l) + \sum_{z=1}^{q}\sum_{l=0}^{n_u} b_{zl}^{(i)} u_z(k-l) + c_{j0}^{(i)} \tag{2}$$

where $j=1,2,\dots r$ is the number of system outputs and $z=1,2\dots q$ is the number of system inputs. The corresponding $i^{th}$ rule from the above fuzzy logic system can be written as follow:

$$R^{(i)} : if\ x_1\ is\ \tilde{A}_1^{(i)}\ and\ x_p\ is\ \tilde{A}_p^{(i)}\ then\ f_j^{(i)}(k) \tag{3}$$

where $x=[x_1,x_2,\dots x_p]$ is the input vector witch represents the input model space and defines the subsets of the elements of the rule premise part and the rule consequent part. The current value of each vector element is equal to a certain previous value of the input or the output of the process. The parameter $f_j$, with $j=1,2,\dots,r$ is the corresponding Sugeno function in notion to $j$-$^{th}$ system output. $\tilde{A}^{(i)}{}_p$ with $p=1,2,3..p$ and $i=1,2,3,\dots,N$ are the Gaussian membership functions with corresponding mean and variance parameters respectively and $y_j$ is the output of $i^{th}$ rule. In this case the number of the fuzzy rules is ensured by combination of $p$ input variables and their $m$ fuzzy sets and it is equal to $N=m^p$. The crisp coefficients $a_{j1},a_{j2},\dots a_{jny}$, $b_{z1}$, $b_{z2},\dots b_{znu}$ are the coefficients into the Sugeno linear function $f_j$ and they correspond to a local discrete transfer function of the model.

The fuzzy inference should match the output of the fuzzifier with the fuzzy logic rules performing fuzzy implication and approximation reasoning to decide the value of the modeled output variable. Fuzzy implication is realized by means of the product composition:

$$\mu_{yj}^{(i)} = \mu_{1d}^{(i)} * \mu_{2d}^{(i)} * \dots * \mu_{sd}^{(i)} \tag{4}$$

where $\mu_{sd}^{(i)}$ specify the membership degree upon fired $d^{th}$ fuzzy set of the corresponded $s^{-th}$ input signal and calculated according to a Gaussian membership function.The exact value of $s$ depends on the system configuration and the number of process inputs and outputs. In general $s$ represents the size of the input model space regarding the process variables and its relations. In case of its equal number *r=q*, *s=2(r+1).*

The modeled process output parameter, after defyzzification is expressed as a weighted mean value. For model predictive control applications the more useful form of the model output computation for whole number *N* of rules at the moment *k* is the equation:

$$\hat{y}_{M_j}(\kappa) = \sum_{i=1}^{N} f_j^{(i)}(k)\,\overline{\mu}_{y_j}^{(i)}(k) \tag{5}$$

where $\overline{\mu}_{y_j}^{(i)}$ is the normalized value of the membership function degree.

*2.1. Identification of the fuzzy predictive model*

A simplified fuzzy-neural approach is applied in this work, because of its simplicity and recurrent implementation of the tuning procedure for on-line applications. A fuzzy-neural network is proposed for this purpose as well as an application of a two steps simplified gradient descent algorithm for adaptive tuning. On the Fig. 1 is presented such a structure, where $x_1, x_2, \ldots x_p$ are the input nodes in first layer connected to the fuzzyfication $\mu$ - modules in the second layer. The rules *R* – modules from third layer interpret the rules and give their outputs to the $\mu_y$ – modules in the fourth layer related to the output parameter $y_j$, which is formed by the output *Y* – node in the fifth layer. The nodes in the layer two are term nodes, which acts as membership functions to present the terms of the respective linguistic variables. This structure enables adaptation of the model properties according to the changes of the process parameters and contents of nonlinearity.

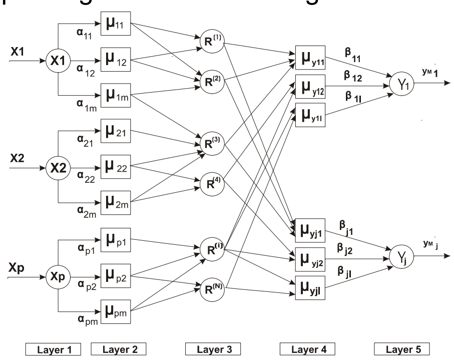


Fig. 1 The structure of the proposed fuzzy-neural network

*2.2 Learning algorithm for the designed MIMO fuzzy neural model*

It is used two steps gradient descent learning procedure [2] as a learning algorithm of the internal fuzzy neural model. It is based on the minimization of an instant error between the process output and the model output. It is need to be adjusted two groups of parameters in the fuzzy neural architecture – premise and consequent parameters. The consequent parameters are the crisp coefficients into the Sugeno function $f_j$ and they are calculated as shown below:

$$\beta_{0jl}(k+1) = \beta_{0jl}(k) + \eta(y_j - y_{M_j})\overline{\mu}_{yj}^{(i)}(k); \qquad \beta_{jl}(k+1) = \beta_{jl}(k) + \eta(y_j - y_{M_j})\overline{\mu}_{yj}^{(i)}(k)x_p(k) \tag{6}$$

in which $\eta$ is the learning rate and $\beta_{jl}$ is an adjustable $l^{th}$ coefficient ($a_{jl}$ or $b_{zl}$) in the Sugeno function $f_j$ of the $i^{th}$ activated rule. The premise parameters are the centre $c_{pm}$ and the deviation $\sigma_{pm}$ of a fuzzy set. They can be calculated using the following equations:

$$c_{pm}(k+1) = c_{pm}(k) + \eta(y_j - y_{M_j})\bar{\mu}^{(i)}_{yj}(k)[f_j^{(i)} - \hat{y}_{M_j}(k)]\frac{[x_p(k)-c_{pm}(k)]}{c^2_{pm}(k)} \qquad (7)$$

$$\sigma_{pm}(k+1) = \sigma_{pm}(k) + \eta(y_l - y_{M_j})\bar{\mu}^{(i)}_{yj}(k)[f_j^{(i)} - \hat{y}_{M_j}(k)]\frac{[x_p(k)-\sigma_{pm}(k)]}{\sigma^2_{pm}(k)} \qquad (8)$$

## 3. BASICS OF MODEL PREDICTIVE CONTROL STRATEGY

Nonlinear Model Predictive Control (NMPC) as it was applied with the MIMO Takagi-Sugeno fuzzy-neural process model can be described in general with a block diagram, as it is depicted in Figure 2

Using the Takagi-Sugeno fuzzy neural model, the Optimization algorithm computes the future control actions at each sampling period, by minimizing the following cost function:

$$J(k,u(k)) = \sum_{i=N_1}^{N_2}(r_1(k+i)-\hat{y}_{M_1}(k+i))^2 + \rho_1\sum_{i=1}^{N_u}\Delta u_1(k+i-1)^2 + \ldots + \sum_{i=N_1}^{N_2}(r_j(k+i)-\hat{y}_{M_j}(k+i))^2 + \rho_z\sum_{i=1}^{N_u}\Delta u_z(k+i-1)^2 \qquad (9)$$

where $\hat{y}_{M_j}$ is the predicted model output, $r_j$ is the $j^{-th}$ reference signal and $u_z$ is the $z^{-th}$ control action. The tuning parameters of the predictive controller are: $N_1$, $N_2$, $N_u$ and $\rho_z$. $N_1$ is the minimum prediction horizon, $N_2$ is the maximum prediction horizon, $N_u$ is the control horizon and $\rho_z$ is the weighting factor penalizing changes in the control actions.

When the criterion function is a quadratic one and there are no constraints on the control action, the cost function can be minimized analytically. If the criterion $J$ is minimized with respect to the future control actions $u$, then their optimal values can be calculated by applying the condition:

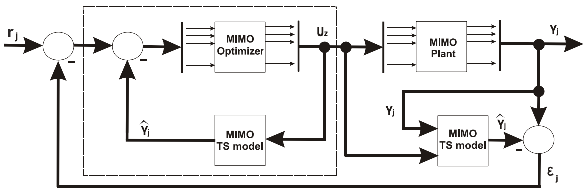


Fig.2 Block diagram of MIMO model predictive control system

$$\nabla J[k,U_z(k)] = \left[\frac{\partial J[k,U_z(k)]}{\partial u_z(k)}, \frac{\partial J[k,U_z(k)]}{\partial u_z(k+1)}, \ldots, \frac{\partial J[k,U_z(k)]}{\partial u_z(k+N_{u_z}-1)}\right] = 0 \qquad (10)$$

## 4. TWO-DIMENSIONAL FUZZY NEURAL NONLINEAR MODEL PREDICTIVE CONTROL

In this paper it is presented a case of multivariable model predictive control of nonlinear process - quadruple water tank system. The aim of the control is to maintain the level of two of them on the basis of two control signals – the input voltages to the pumps. This is the reason for designing of two-dimensional fuzzy neural predictive controller. So, using the above marks the number of inputs is equal to the number of outputs – $r = q = 2$. In this case it is designed a MIMO Takagi-Sugeno fuzzy neural model with input model space equal to eight and two outputs. In this way the used regression vector is:

$$x(k) = [y_1(k-1), y_1(k-2), u_1(k), u_1(k-1), y_2(k-1), y_2(k-2), u_2(k), u_2(k-1)] \qquad (11)$$

In this way the Sugeno function in notion of each output has the following form:

$$f_j^{(i)}(k) = a_{j1}^{(i)} y_j(k-1) + a_{j2}^{(i)} y_j(k-2) + + b_{10}^{(i)} u_1(k) + b_{11}^{(i)} u_1(k-1) + + b_{20}^{(i)} u_2(k) + b_{21}^{(i)} u_2(k-1) + c_{j0}^{(i)} \quad (12)$$

It can be seen from the above equation (16) that in this case the influence between system outputs is not considered. The reason to simplify this expression is to reduce the computational effort decreasing the number of the fuzzy rules. In this case the fuzzy model is designed to work with 729 fuzzy rules. It is well known that using any supplementary relations leads to larger number of fuzzy rules, e.i. tremendous computational efforts.

The second part in NMPC scheme is the MIMO Optimizer which computes a sequence of optimal control actions for each manipulated input variable at each sampling period. In this case the used optimization criterion is:

$$J(k,u(k)) = \sum_{i=N_1}^{N_2} (r_1(k+i) - \hat{y}_{M_1}(k+i))^2 + \rho_1 \sum_{i=1}^{N_u} \Delta u_1(k+i-1)^2 + \sum_{i=N_1}^{N_2} (r_2(k+i) - \hat{y}_{M_2}(k+i))^2 + \rho_2 \sum_{i=1}^{N_u} \Delta u_2(k+i-1)^2 \quad (13)$$

The main tuning parameters of the predictive controller are set to: $N_1=1$; $N_2=5$; $N_u=2$.

## 5. SIMULATION RESULTS

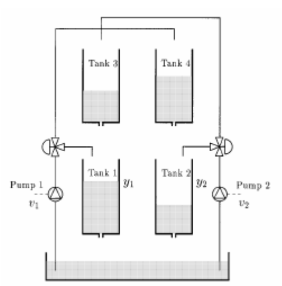


Fig.3 The quadruple-tank process

To prove the efficiency of two-dimensional fuzzy neural nonlinear model predictive controller there are implemented some simulation experiments. As a plant it is used a quadruple water tank system. A schematic diagram of the process is shown in Fig.3. The target is to control the level in the lower two tanks by two pumps. The process inputs are $v_1$ and $v_2$ (input voltages to the pumps) and the outputs are $y_1$ and $y_2$ (voltages from level measurement devices). More detailed description of the quadruple water tank system can be find in [3].

### *5.1 Experimental results*

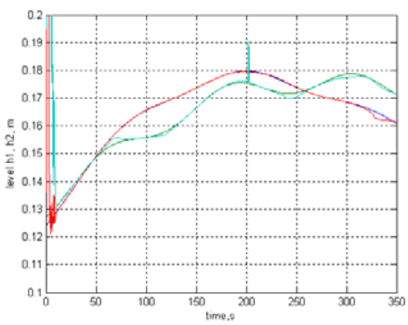


Fig. 4 Training of the MIMO TS model with sinusoidal signal

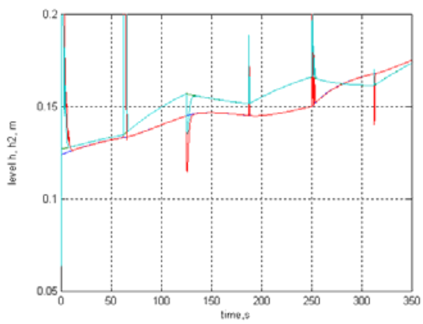


Fig. 5 Training of the MIMO TS model with squared signal

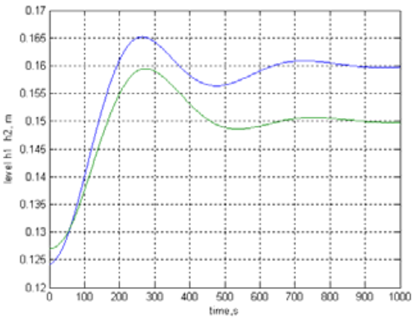


Fig. 7 GPC level control
at $\rho_{1,2}$=0.005

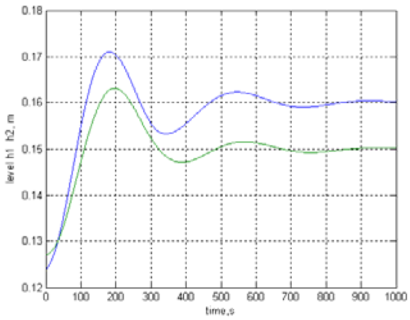


Fig. 8 GPC level control
at $\rho_{1,2}$=0.008

## 6. CONCLUSIONS

In this paper is presented a method for designing a multivariable nonlinear predictive controller based on a Takagi-Sugeno fuzzy-neural MIMO model. It is used a simplified gradient technique in the optimization task to calculate predictions on the future control actions and for on-line adaptation of the fuzzy-neural model. The simulation results show the efficiency of the designed MIMO fuzzy neural predictive controller. The experiments are made with two-dimensional predictive controller. The proposed approach can be easily extended to control different configurations of system variables.

## 7. REFERENCES

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