SUPERVISORY TUNING OF NONLINEAR MODEL PREDICTIVE CONTROLLER

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Abstract: It is proposed in this work a fuzzy supervisory algorithm for adaptive tuning of a weighting factor in Nonlinear Model Predictive Control (NMPC). A nonlinear predictive controller is designed on the basis of the Takagi-Sugeno neuro fuzzy predictive model. A high control performance can be obtained, by on-line adaptation of the tuned parameter. The effectiveness of the proposed approach is demonstrated by experimental simulations in *MATLAB & SIMULINK* environment to level control of a three cascaded water tanks. The simulation results prove the efficiency of the proposed algorithm and show improvements in some quality control criteria.

Keywords**:** Nonlinear Predictive Control, Adaptive Control, Takagi-Sugeno Fuzzy Model, Supervisory Tuning

1. INTRODUCUTION

Model Predictive Control (MPC) has emerged as one of the most attractive control technique during the past decade. In MPC a process dynamic model is used to predict future outputs over a prescribed period called prediction horizon. Some of the most applied predictive control algorithms in various industrial processes are: Dynamic Matrix Control (DMC) [2], Generalised Predictive Control (GPC) and Model Algorithmic Control (MAC) [3].

In general, a wide range of industrial processes is inherently nonlinear. For such nonlinear systems, a linear MPC algorithm may not insure satisfactory dynamic performance. Recently, several researchers have developed nonlinear model predictive control (NMPC) algorithms that work with different types of nonlinear models. These models can be divided into two classes. The first one includes models based on fundamental relationships (first principle). Such models can be accurate over a wide range of operating conditions, but they are very difficult to develop for many industrial cases. On the other point of view, these models may require tremendous computational effort for optimization and make them unsuitable for on-line applications. The second class models are based on empirical data, such as artificial neural networks and fuzzy logic models. Typical for this group is a precise description of the process by a set of linear submodels. In this way the design of a model predictive controller can be greatly simplified.

The Takagi-Sugeno model is a quasi-linear empirical model developed by means of fuzzy logic for each local subsystem. The whole process behaviour is characterized by weighted sum of the outputs from all quasi-linear fuzzy implications. The main advantage of the Takagi-Sugeno model is the soft transition through any operating regions.

In classical predictive control scheme the model outputs are used to compute the future control actions by minimizing a cost function, over the prediction horizon. The good system performance depends on model accuracy and parameters in the objective function – weighting factors, prediction and control horizons. In practice, there is no systematic way to define these parameters and they should be selected heuristically, by “trial and error” procedures based on simulations results. In this way finding proper parameters could be a long and inaccurate process. For this purpose it is needed to be developed a tuning algorithm that can provide a satisfactory system performance.

It is presented in this paper a nonlinear predictive control strategy based on a Takagi-Sugeno fuzzy-neural model. The predictive control scheme is modified by additional supervisory level for adaptive tuning of a weighting factor in optimization criterion. The proposed approach is studied by experimental simulations in *Matlab* environment to control the level of a three cascaded water tanks.

2. BASICS OF MODEL PREDICTIVE CONTROL STRATEGY

Model predictive control (MPC) is a common name for computer control algorithms that use an explicit process model to predict the future plant response. According to this chosen period, also known as a prediction horizon, the MPC algorithm optimizes the manipulated variable to obtain an optimal future plant response. The input of chosen length, also known as a control horizon, is sent to the plant and then the entire sequence is repeated again in the next time period.

Nonlinear Model Predictive Control (NMPC) as it was applied with the Takagi-Sugeno fuzzy- neural process model can be described in general with a block diagram, as it is depicted in the Figure 1.

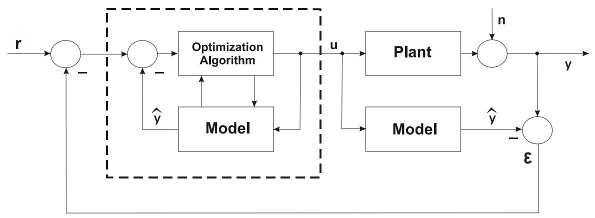

Fig.1 Block diagram of the model predictive control system

The Takagi-Sugeno fuzzy-neural models are suitable to model a class of nonlinear systems. As it is well known a wide class of nonlinear dynamic systems can be described in discrete time by the NARX (Nonlinear AutoregRessive model with eXogenous inputs) input-output model. The used model in this paper is also taken in the NARX type:

$$y(k)=f_y(x(k)) \qquad (1)$$

where the unknown nonlinear function f_y can be approximated by the Takagi-Sugeno type fuzzy rules:

$$R^{(i)}: if\ x_1\ is\ \widetilde{A}_1^{(i)}\ and\ x_p\ is\ \widetilde{A}_p^{(i)}\ then\ f_y^{(i)}(k) \qquad (2)$$

$$\begin{aligned} f_y^{(i)}(k) &= a_1^{(i)}y(k-1)+a_2^{(i)}y(k-2)+\cdots+a_{ny}^{(i)}y(k-n_y) \\ &+b_1^{(i)}u(k)+b_2^{(i)}u(k-1)+\cdots+b_{nu}^{(i)}u(k-n_u)+c_0^{(i)} \end{aligned} \qquad (3)$$

$(i)=1,2,...N$, where N is the number of the fuzzy sets, A_i is an activated fuzzy set defined in the universe of discourse of the input x_i and the crisp coefficients a_1, a_2,...a_{ny}, b_1, b_2,...b_{nu} are the coefficients into the Sugeno function f_y.

In the Takagi-Sugeno fuzzy model it is needed to determine unknown parameters – number of membership function, their shape and the parameters of the function f_y in the consequent part of the rules. This is an identification procedure for which have been proposed numerous approaches [6]. It is applied a simplified fuzzy neural approach in this work.

2.1 Learning algorithm for the designed fuzzy neural model

It is used two steps gradient learning procedure [?] as a learning algorithm of the internal fuzzy neural model. The algorithm is based on the minimization of an instant error between the process output and the model output. It is need to be adjusted two groups of parameters in the fuzzy neural architecture – premise and consequent parameters. The consequent parameters are the coefficients a_1, a_2,...a_{ny}, b_1, b_2,...b_{nu} in the Sugeno function f_y and they are calculated firstly by following equations:

$$\beta_{ij}(k+1)=\beta_{ij}(k)+\eta(y-y_M)\overline{\mu}_y^{(j)}(k)x_i(k) \qquad (4)$$

$$\beta_{0j}(k+1)=\beta_{0j}(k)+\eta(y-y_M)\overline{\mu}_y^{(j)}(k) \qquad (5)$$

in which η is the learning rate and β_{ij} is an adjustable i^{-th} coefficient (a_i or b_i) in the Sugeno function f_y of the j^{-th} activated rule.

The premise parameters are the centre c_{ij} and the deviation σ_{ij} of each fuzzy set. They can be calculated using the following equations:

$$c_{ij}(k+1)=c_{ij}(k)+\eta(y-y_M)\overline{\mu}_y^{(j)}(k)[f_y^{(i)}-\hat{y}(k)]\frac{[x_i(k)-c_{ij}(k)]}{c_{ij}^2(k)} \qquad (6)$$

$$\sigma_{ij}(k+1)=\sigma_{ij}(k)+\eta(y-y_M)\overline{\mu}_y^{(j)}(k)[f_y^{(i)}-\hat{y}(k)]\frac{[x_i(k)-\sigma_{ij}(k)]}{\sigma_{ij}^2(k)} \qquad (7)$$

in which c_{ij} and σ_{ij} are the centre and the deviation of a corresponding fuzzy set.

Using the Takagi-Sugeno fuzzy neural model, the Optimization Algorithm computes the future control actions at each sampling period, by minimizing the following cost function:

$$J(k,u(k))=\sum_{i=N_1}^{N_2}(r(k+i)-\hat{y}(k+i))^2+\rho\sum_{i=1}^{N_u}\Delta u(k+i-1)^2 \qquad (8)$$

where y is the predicted model output, r is the reference and u is the control action. The tuning parameters of the predictive controller are: N_1, N_2, N_u and ρ. N_1 is minimum prediction horizon, N_2 is maximum prediction horizon, N_u is control horizon and ρ is the weighting factor penalizing changes in the control actions.

When the criterion function is a quadratic one and there are no constraints on the control action, the cost function can be minimized analytically. If the criterion J is minimized with respect to the future control actions u, then their optimal values can be calculated by applying the condition:

$$\nabla J[k,U(k)]=[\frac{\partial J[k,U(k)]}{\partial u(k)},\frac{\partial J[k,U(k)]}{\partial u(k+1)},...,\frac{\partial J[k,U(k)]}{\partial u(k+N_u-1)}]=0 \qquad (9)$$

3. DESIGN OF SUPERVISORY LEVEL

As it was mentioned above the main tuning parameters in the classical predictive controller are: the control and the prediction horizons and the weighting factors. In general, the exact value of each parameter is selected heuristically, regarding the current process. On the other hand it can be seen that these selected values may not provide the desired system performance. In this way to improve the system dynamics it is needed to develop methods for its adaptive tuning. So, here it is proposed an idea for fuzzy adaptive tuning of the weighting factor ρ in optimization criterion (8), maintaining the other parameters as constants.

3.1The structure of the fuzzy supervisory level

The two main parts in the MPC control scheme (Figure 1) are: the process model for prediction of the system output over the prediction horizon and the predictive controller for computation of the future control actions on the basis of predicted system output in each sampling period.

On the other hand the classical control scheme using PID controller is building on different principles. When the PID controller is used, the control action is computed on the basis of the system error, as a difference between the reference and the system output. It is known that this error is a structure parameter which indicates the system dynamics and the aim of PID control is to minimize it. So, it can be seen that the system error is a parameter with strong relation regarding the process. It is used a predicted system error in the MPC scheme to compute the control action.

In this way it was emerged the idea to design a fuzzy supervisor that will tune adaptively the weighting factor ρ with relation to the system error and their first derivative. However, finding a proper mathematical expression between mentioned parameters is a difficult or impossible task. So, the fuzzy logic seems to be a good decision to obtain such relation using simple expert rules. The structure of the designed fuzzy supervisor is shown on the Figure 2.

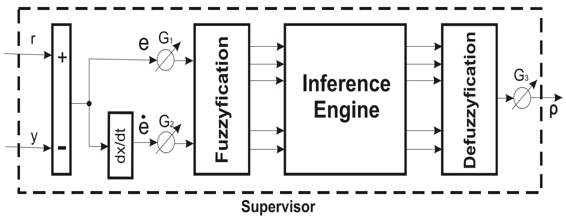

Fig.2 Structure of the fuzzy supervisory level

The modified MPC control scheme with the applied fuzzy supervisor is shown on the Fig.3:

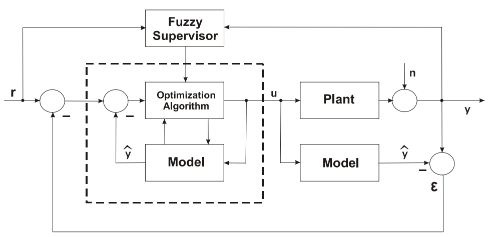

Fig.3 MPC control scheme with fuzzy supervisor

In such a way the modified control scheme as it is shown above, will provide adaptive tuning of the weighting factor ρ at each sampling period.

3.2 The fuzzy supervisor tuning procedure

The main tuning parameters in the structure (Fig. 2) of the fuzzy supervisor are: the shape and the number of the membership functions and the scaling factors of the input and output signals. It is well known that the number of the fuzzy rules in the fuzzy inference engine depends on the number of the membership functions. The formulation of the fuzzy rules applied into the supervisor is in the next form:

$$R^{(i)}: if\ e(k) \in \widetilde{E}_i\ and\ \Delta e(k) \in \Delta\widetilde{E}_j\ then\ \rho(k) \in \widetilde{P}_k \quad (10)$$

where $\widetilde{E}_i, \widetilde{E}_j$ *and* $\widetilde{P}_k$ are the activated fuzzy sets in $R^{(i)}$ rule.

The used look-up table (Table 1) for the fuzzy rules is composed as follow:

Δe \ e	**NL**	**NM**	**NS**	**ZR**	**PS**	**PM**	**PL**
NL	NL	NM	NM	NS	NS	ZR	ZR
NM	NM	NM	NS	NS	ZR	ZR	ZR
NS	NM	NS	NS	ZR	ZR	ZR	PS
ZR	NS	NS	ZR	ZR	ZR	PS	PS
PS	NS	ZR	ZR	ZR	PS	PS	PM
PM	ZR	ZR	ZR	PS	PS	PM	PM
PL	ZR	ZR	PS	PS	PM	PM	PL

Table 1. Look-up table for fuzzy rules

The final deffuzyfication procedure is based on the center of gravity method which gives satisfactory calculation of the crisp output signal value $\rho(k)$.

3.3 The other known tuning procedure

The standard formulation of the cost function (8) is extended by the time dependent penalty factor $\rho(k)=\rho$. For linear processes a constant ρ is usually sufficient, but for nonlinear processes the changing of the process gain must be taken into consideration. In operation points of a relatively slow process gain, larger changes in the manipulated variable can be accepted. In contrast, aggressive control actions are not desirable in regimes of high gains. Therefore, the following equation is recommended for nonlinear processes [7]:

$$\rho(k) = K_p^{\,2}(k)\rho_o \quad (11)$$

where K_p is the current process gain, and ρ_0 is a constant value of the weighting factor.

The current process gain $Kp(k)$ can be determined by dynamic linearization of the fuzzy model [4].

Using Linear Variant Parameters (LVP) interpretation the steady-state of the model can be easily described. This way the gain of the local linear model can be calculated from crisp coefficients of each linear function. Then the global gain of the neuro-fuzzy model can be estimated as a weighted mean value of the local model gains.

4. SIMULATION RESULTS

4.1 Plant description

The controlled process, considered in this work is a plant composed of three water tanks. The plant model is presented at the Figure 4. The main parameters of the process are: input flows Q_1 and Q_2; S – cross-sectional area of the tanks, equal for each one; S_a – outlet area of each tank; $h_{1,2,3}$ – the level in each tank.

The mathematical description of the plant is given bellow by equations (12), (13), (14). The aim of control task is maintaining the level of the third tank h_3 regarding the inflow Q_1 at the constant inflow Q_2.

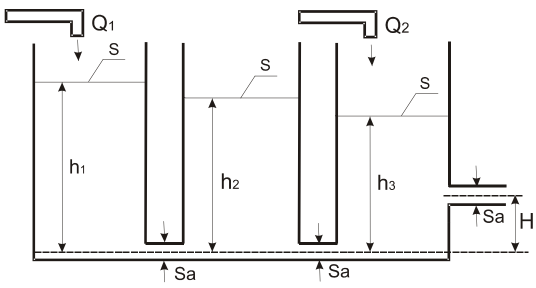

Fig. 4 The model of the three cascaded water tanks

$$\dot{h}_1 = -\frac{S_a}{S}F(h_1 - h_2) + \frac{1}{S}Q_1 \tag{12}$$

$$\dot{h}_2 = -\frac{S_a}{S}F(h_1 - h_2) - \frac{S_a}{S}F(h_2 - h_3) \tag{13}$$

$$\dot{h}_3 = -\frac{S_a}{S}F(h_2 - h_3) - \frac{S_a}{S}F(h_3 - H) + \frac{1}{S}Q_2 \tag{14}$$

4.2 Initial conditions for simulation

The simulation results are obtained with the next initial conditions:

- $N_1=1,\ N_2=5,\ N_u=3;$
- *Reference r=0.5 m;*
- *Variable reference r=0.5m t=[0:1000]; r=0.6m t=[1000:2000]; r=0.55 t=[2000:3000].*

4.3 Quality control criteria

The chosen quality criteria in the control system are:

- *System Overshoot - σ [%]*
- *Root Mean Squared Error*

$$RMSE = \frac{1}{N}\sqrt{\sum_{k=1}^{N}(r(k) - y(k))^2} \tag{15}$$

4.4 Experimental results

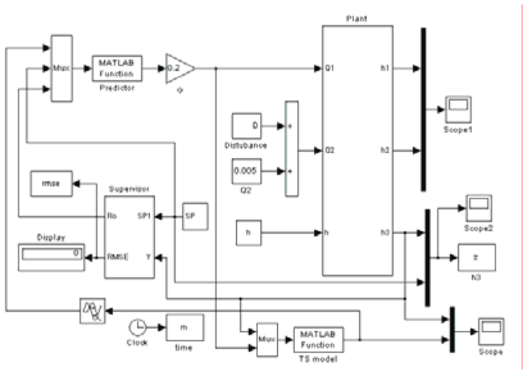

Fig.5 Control scheme with supervisory level in SIMULINK implementation

4.4.1 Results with classical GPC algorithm

It is shown at the Fig. 6 the transient responses with different constant values of ρ. It is needed to obtain the range of values that are suitable for the studied nonlinear process. It is evident that the lower values of ρ provide a slow process response with a large settling time and small values of overshoot (Table 2). After increasing, the value of ρ gives inverse results in notion of quality criteria. The most appropriate value of parameter ρ in this case is $\rho=0.0003$.

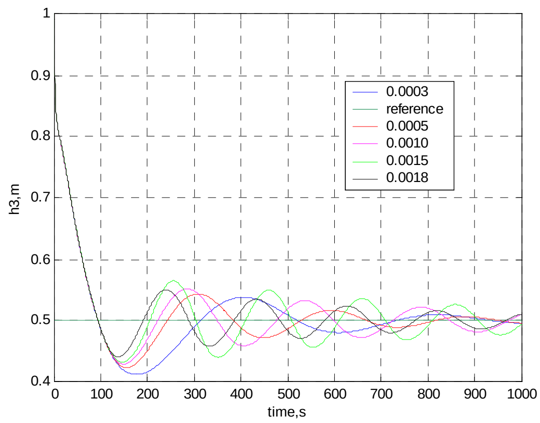

Fig. 6 Transient responses with different values of ρ

ρ	*σ [%]*	*RMSE*
0.0003	38	0.0693
0.0005	56	0.0664
0.0010	73	0.0672
0.0015	97	0.0705
0.0018	99	0.0657

Table 2

4.4.2 Results with tuning of ρ regardingto the process gain

Using equation (11) and on the basis of the previous results for the range of ρ, they are realized simulation experiments for two initial values which are denoted with ρ_0 (Table 3) . The experimental results are depicted at the Figure (7).

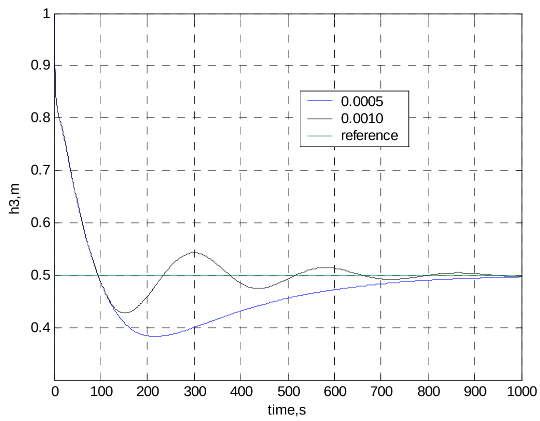

Fig. 7 Transient processes with step calculation of ρ

ρ₀	*σ [%]*	*RMSE*
0.0005	0	0.0826
0.0010	58	0.0657

Table 3

4.4.3 Results with supervisory tuning of ρ

In the beginning the fuzzy supervisor was designed to work with three Gaussian membership functions with respect to each input and output variable. It is provide experiments with different number and shape of membership functions to explore the influence of these parameters. It is shown at the figure 8 that after increasing of the number of the membership functions the system dynamics is improved.

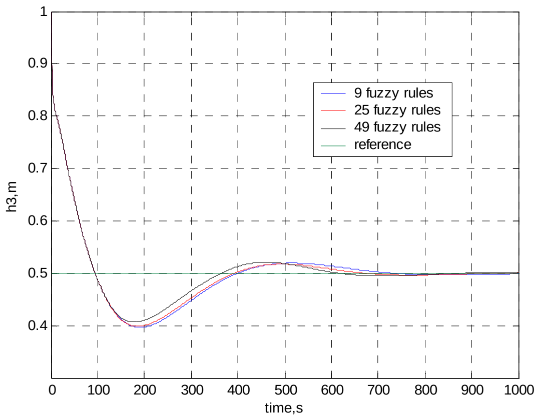

Fig. 8 Supervisory tuning by using of different number of fuzzy rules

When triangular membership functions are chosen for fuzzyfication the given results are almost the same, but with a larger overshoot and a RMSE (Fig. 9), (Table 4).

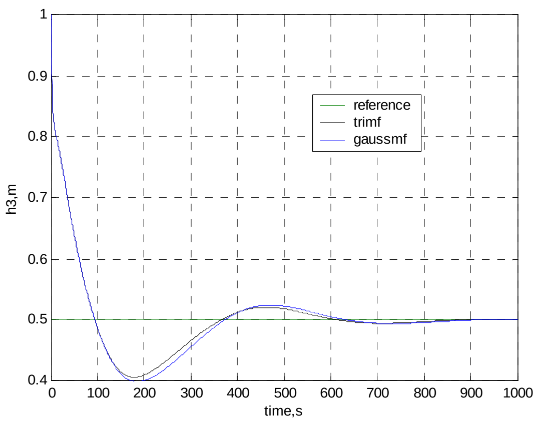

Fig. 9 Supervisory tuning by varying the shape of the membership functions

ρ_0 =0.001		
Gaussian membership functions		
Fuzzy rules	*σ [%]*	*RMSE*
9	19	0.0724
25	22	0.0718
49	23	0.0698
Triangular membership functions		
25	25	0.0734

Table 4

They are also made simulations with a large number of Gaussian membership functions varying the initial value ρ_0. The results show that after increasing the initial value ρ_0, the predictive controller ensures faster system dynamic (Fig. 10), (Table 5).

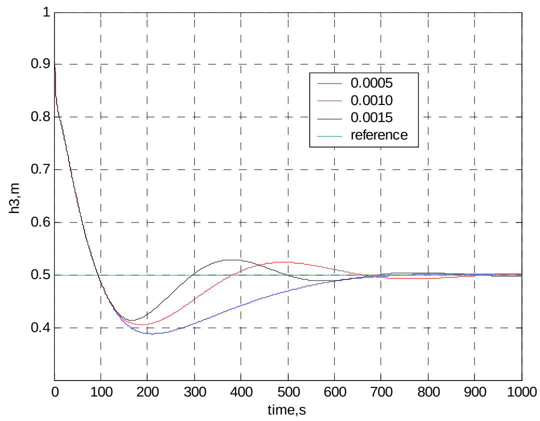

Fig. 10 Supervisory tuning by varying the initial value of ρ

Fuzzy rules 49		
Gaussian membership functions		
ρ	*σ [%]*	*RMSE*
0.0005	0	0.0802
0.0010	22	0.0698
0.0015	32	0.0676

Table 5

4.4.4 Comparison between different tuning methods

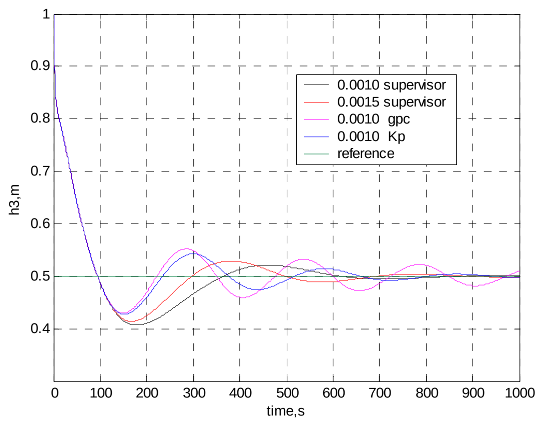

Fig. 11 Comparison between different tuning methods

The comparison between different tuning algorithms, in notion of the other methods, shows the advantage of the control scheme using the fuzzy supervisor. The main disadvantage of the supervisory control system is the additional computational effort. It can be overcome with increasing of the initial value of weighting factor.

4.5 Comparison between different tuning methods in case of a variable system reference.

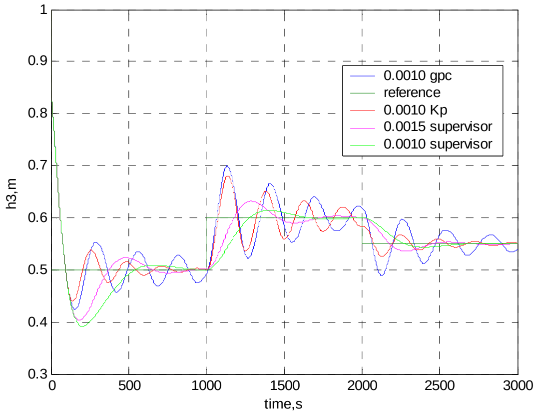

Fig. 12 Comparison between different tuning methods

Tuning Strategy	ρ_0	*Reference [m]*			*RMSE*
		0.5	*0.6*	*0.55*	
		σ [%]	*σ [%]*	*σ [%]*	
GPC	0.0010	73	77	78	0.04549
Kp	0.0010	58	79	72	0.04724
Sup.	0.0010	22	20	17	0.04962
Sup.	0.0015	32	31	28	0.04378

Table 6

4.6 System dynamics in presence of an additive disturbance

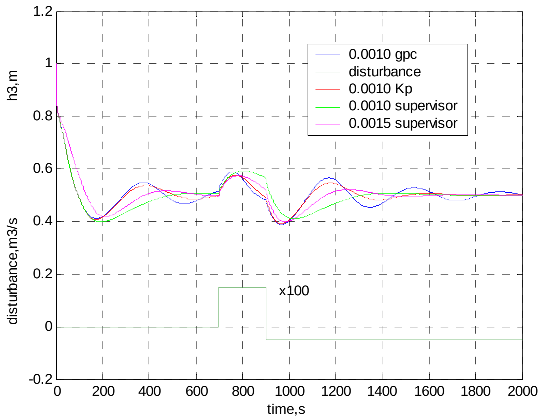

Fig. 13 Comparison between different tuning methods in presence of a disturbance

5. CONCLUSIONS

A fuzzy supervisory algorithm for adaptive tuning of a weighting factor in NMPC is presented for the control of the level of three cascaded water tanks. It is made an experimental simulations in the Simulink environment with different tuning methods. The obtained results in this work show that GPC control algorithm with constant weighting factor ρ can not ensure the desired quality of system performance. It is also investigated how this factor can be calculated simultaneously on the base of the process gain K_p. This method for weighting factor estimation gives an improvement of the system dynamics regarding the quality control criteria, but there are still a higher values of the overshoot.

As additional part in the classical control scheme the proposed supervisory level gives better results. The quality control criteria attempts its desired values. On the other hand the supervisory level can provide needed system performance also in case of variable reference signals and in presence of an additive disturbance. Using the triangular membership functions aggravates the quality criteria. On the other hand the use of the Gaussian membership functions is recommended for filtering of additive disturbances. Increased number of the fuzzy rules in the supervisory structure improves the system dynamics on respect of the quality criteria. The simulation results prove the efficiency of the proposed algorithm and show improvements in some quality control criteria.

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